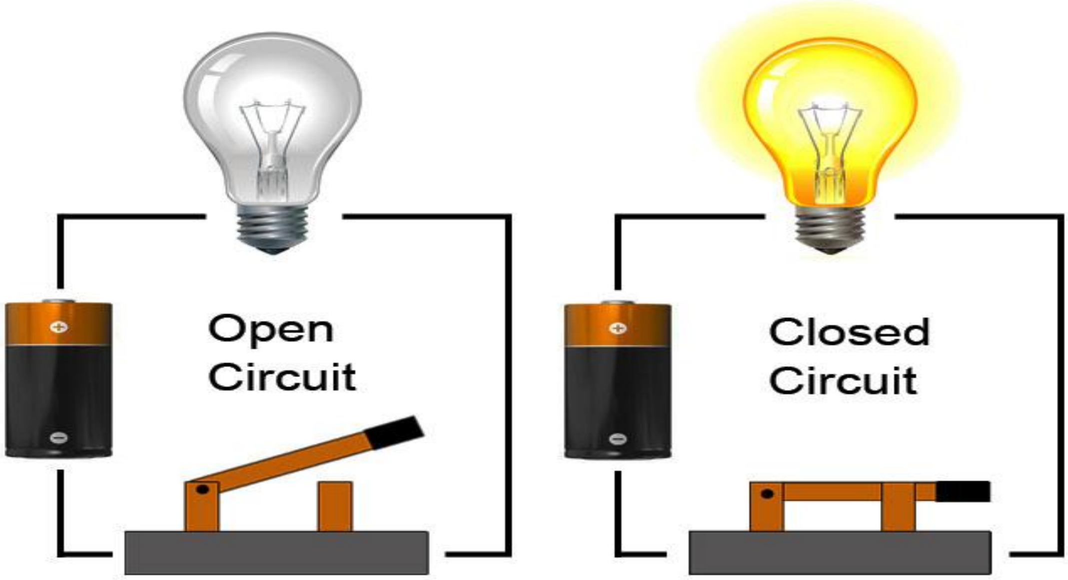
BHADRAK ENGINEERING SCHOOL & TECHNOLOGY (BEST), ASURALI, BHADRAK

Circuit & Network Theory

(Th.- 02)

(As per the 2019-20 syllabus of the SCTE&VT, Bhubaneswar, Odisha)



Third Semester

Electrical Engg.

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CIRCUIT & NETWORK THEORY
CHAPTER-WISE DISTRIBUTION OF PERIODS & MARKS

Sl. No.	Chapter No.	Name of the Chapter	Periods as per Syllabus	Expected Marks
01	01	Magnetic Circuits	07	10
02	02	Coupled Circuits	05	15
03	03	Circuit Elements And Analysis	06	12
04	04	Network Theorems	08	15
05	05	Ac Circuit And Resonance	08	16
06	06	Poly-phase Circuit	06	10
07	07	Transients	06	10
08	08	Two-Port Network	08	12
09	09	Filters	06	10
TOTAL			**60**	**110**

CHAPTER NO.- 03
CIRCUIT ELEMENTS AND ANALYSIS

Learning objective:

3.1 Active, Passive, Unilateral & bilateral, Linear & Nonlinear elements
3.2 Mesh Analysis, Mesh Equations by inspection
3 .3 Super mesh Analysis
3.4 Nodal Analysis, Nodal Equations by inspection
3. 5 Super node Analysis.
3. 6 Source Transformation Technique
3.7 Solve numerical problems (With Independent Sources Only)

Electricity:

- It is defined as a source of energy which can not seen but its effect can easily felt.

Charge(Q):

- It is defined as the deficiency & excess of electrons of an atom of the material.
- There are two types of electrical charges as positive charge(+Q) and negative charge (-Q).
- It is denoted as q or Q and its unit is coulomb.

Current(I):

- The rate of flow of charge from one point to another point in a conductor is called as electric current.
- It is denoted as I and its unit is ampere.

Mathematically, $I = \frac{dQ}{dt}$

Voltage(V):

- It is defined as the amount of work required to bring a unit positive charge from infinite distance to the point of observation on consideration.
- It is also known as electric potential or electric pressure.
- It is denoted by V and its unit is $\frac{Joule}{Coulomb} = volt.$

Mathematically, $V = \frac{w}{Q}$

Power:

- It is defined as the rate of doing work is called as power.
- It is denoted as P and its unit is watt.

Mathematically, $P = \frac{w}{t} = \frac{work}{time}, \frac{Joule}{second} = watt$

We know that, $V = \frac{w}{Q}$

Or $w = Vq$

Dividing time in both sides.

$$\frac{w}{t} = \frac{Vq}{t}$$

Or $Power(P) = \frac{Vq}{t} = V \times I$

Energy(E):

- The capacity to do work is called as energy.
- Energy is equal to the product of Power and time.

$Electrical\ Energy = Power \times time$

$$E = P \times t = V \times I \times t = I^2Rt = \frac{V^2}{R}t$$

- It's unit is Watt-hour or KWh.

Ohm's Law:

Statement:

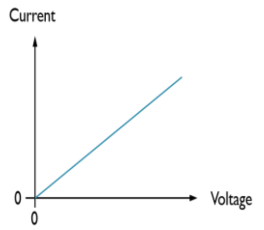
It states that" At constant temperature, the current(I) flowing through a metallic conductor is directly proportional to the potential difference(V) between the two ends of the conductor ".

Mathematically : $I \alpha V$

Or $I = \frac{V}{R}$ or V=IR

Where R is a constant of proportionality and is called resistance of the conductor.

The V-I relation for resistor according to Ohm's law is



Resistance(R):

- The physical property of a material to oppose the flow of current, is known as resistance.
- It is represented by the symbol R.
- The Resistance is measured in ohms (Ω).

Mathematically, $R = \rho \frac{L}{A}$

Where R= Resistance of the conductor.

L= Length of the conductor.

A= Cross sectional area of the conductor

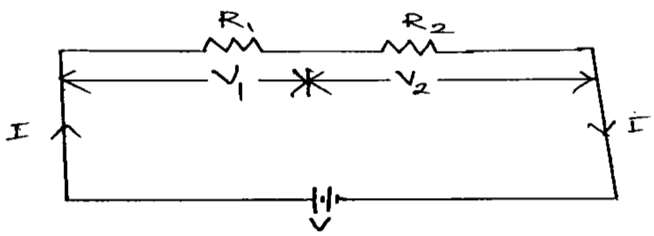
Conductance(G):

- The physical property of a material to allow the flow of current, is known as conductance.
- It is the reciprocal of resistance.
- It is represented by the symbol G.

Mathematically $G = \frac{1}{R}$

Resistors are connected in Series:

- The resistors are said to be connected in series ,if they are joined cascaded or end –on –end.



The two resistors are in series, since the same current i flow in both of them.

Applying Ohm's law to each of the resistors, we obtain V_1=iR_1, V_2= iR_2----------(1)

Here total voltage becomes

$V = V_1 + V_2$ -- - - - - -- - -- (2)

Combining equation(1) and (2), we get

$V = V_1 + V_2 = IR_1 + IR_2 = I(R_1+R_2)$ - --- - - - -- (3)

Equation (3) can be written as $V = IR_{eq}$---------*(4)*

Where $R_{eq} = R_1 + R_2$ i.e. the summation of two resistors.

From equation(3), we get i= $\frac{V}{R_{eq}} = \frac{V}{R_1+R_2}$ - - - - - -(5)

In general, the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

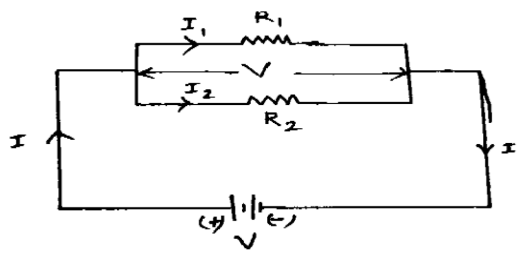
Voltage Division Rule:

To determine the voltage across each resistor in above fig, we substitute Eq. (5) in to Eq. (1) and obtain

$V_1 = \frac{V}{R_1+R_2} R_1$ and $V_2 = \frac{V}{R_1+R_2} R_2$

Resistors are connected in Parallel:

- The resistors are said to be connected in parallel, if the starting points of all resistors are connected in one point and finishing points are connected in one point .



Where two resistors are connected in parallel andtherefore have the same voltage across them.

$V = I_1R_1 = I_2R_2$ - - - - --- - --(1)

$i_1 = \frac{V}{R1}$ and $i_2 = \frac{V}{R2}$ -- - -- - - -- (2)

Here total current ,

$i = i_1 + i_2$ ------------(3)

Substituting Equation 2 into 3, we get

$i = \frac{V}{R_1} + \frac{V}{R_2} = V(\frac{1}{R_1} + \frac{1}{R_2}) = \frac{V}{R_{eq}}$ - - -- - - - (4)

Where R_{eq} is the equivalent resistance of the resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

If a circuit with N resistors in parallel then the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + - -- - + \frac{1}{R_N}$$

Division of current in parallel circuit:

Let,

V= Supply voltage in V.

I= Circuit current in A.

I_1= Branch current at R_1 in A.

I_2= Branch Current at R_2 in A.

We know that the equivalent resistor has the same voltage.

$V= I\,R_{eq.} = I \times \frac{R_1 R_2}{R_1+R_2}$ - - -- - - - -- - - -(5)

Substituting equation (5) in equation(2) we get

$I_1= \frac{I \times R_2}{R_1+R_2}$ and $I_2= \frac{I \times R_1}{R_1+R_2}$

Inductance:

- It is defined as the property of a coil or inductor which opposes the sudden decay or rise of electric current.
- It is denoted as L and its unit is Henery.

Inductors are connected in series and parallel:

- If three inductors L_1,L_2,L_3 are connected in series then the equivalent inductance are can be represented by $L= L_1+L_2+L_3$.
- If three inductors L_1,L_2,L_3 are connected in parallel then the equivalent inductance are can be represented by $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

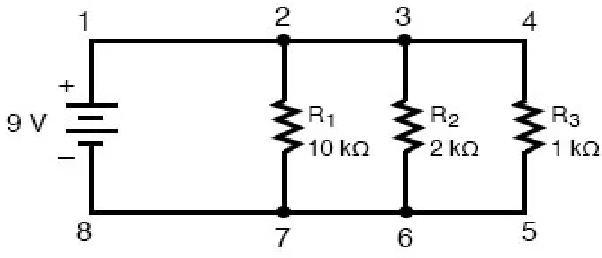
Capacitance:

- It is defined as the property of an element to store the electrical energy.
- It is denoted as C and its unit is Farad.

Capacitors are connected in series and parallel:

- If three capacitors C_1,C_2,C_3 are connected in series then the equivalent capacitance are can be represented by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
- If three capacitors C_1,C_2,C_3 are connected in parallel then the equivalent capacitance are can be represented by $C= C_1+C_2+C_3$.

Q.1 Find the current I passing through and the current passing through each of the resistors in the circuit below.



Solution:

Equivalent resistance, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$= \frac{1}{10\times10^3} + \frac{1}{2\times10^3} + \frac{1}{1\times10^3}$

$= 0.0016$

R_{eq} = 625 ohm.

$I = V/R_{eq}$ = 9/625 = 0.0144, A =14.4 mA

$I_{R1} = V/R_1 = 9/\ 10 \times 10^3 = 0.9$ mA

$I_{R2} = V/R_2 = 2/\ 10 \times 10^3 = 4.5$ mA

$I_{R3} = V/R_3 = 1/\ 10 \times 10^3 = 9$ mA

Some terms are used in network theory:

Circuit:

- A circuit is a float conducting path or closed path through which an electric current either flows or desire to flow.

Parameter:

- The various elements of an electric circuit are called its parameters like resistor, inductor, capacitor, voltage source etc.

Node:

- It is an equipotential point where two or more circuit elements are connected together.

Branch:

- It is the part of the network which lies in between two nodes.

Loop:

- It is a closed path in which no element or node is encountered more than ones.

Mesh:

- It is a loop that contents no other loop within it.

3.1 Active, Passive, Unilateral & bilateral, Linear & Nonlinear elements:

Active element:

- A circuit element is said to be active if it has the ability to supply energy to activate a circuit.
- Example : Voltage source, Current source.

Passive element:

- A circuit element is said to be passive if it absorbs the electrical energy.
- Example : Resistor, Inductor, Capacitor.

Bi-lateral element:

- A circuit element is said to be bi-lateral if the magnitude of current remains same even if the applied voltage polarity is changed.
- Example : Resistor, Inductor, Capacitor.

Uni-lateral element:

- A circuit element is said to be uni-lateral if the magnitude of current passing through an element is affected due to the change in polarity of voltage source.
- Example : All semiconductor device like diode, transistor etc.

Linear element:

- The circuit element which shows linear characteristics of voltage verses current are called as linear element.
- The circuit element which obeys the Ohms law is called as linear element.
- Example : Resistor, Inductor, Capacitor.

Non-Linear element:

- The circuit element in which the current passing through it does not change linearly with linear change in applied voltage is called as non-linear element.
- The circuit element which does not obey the Ohms law is called as non-linear element.
- Example : All semiconductor device like diode, transistor etc.

Kirchhoff's Law:

There are two types of Kirchhoff´s law

i. Kirchhoff´s current law (KCL) or point law
ii. Kirchhoff´s voltage law (KVL) or mesh law

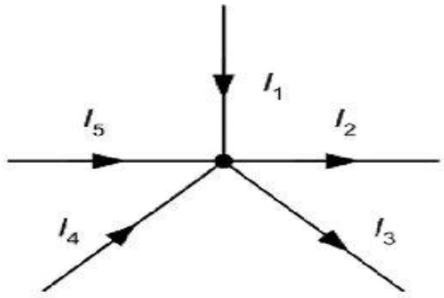
Kirchhoff´s Current law (KCL) or point law:

Statement:

" The algebraic sum of currents meeting at a point or junction is zero“ .

Mathematically:

$$\sum_{i=1}^{n} I = 0$$



Sign convention :

- Incoming current are taken as + ve(positive)
- Outgoing current are taken as – ve (negative)

In the above figure,

I_1, I_4 and I_5 are incoming currents.

I_2 and I_3 are outgoing currents.

According to KCL,

$$+I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

$$\Rightarrow +I_1 + I_4 + I_5 = I_2 + I_3$$

Hence Algebraic sum of currents entering a node=Algebraic sum of current leaving a node.

Kirchhoff´s voltage law (KVL) or mesh law:

Statement:

“The algebraic sum of emf ´s and potential drops across resistors in a closed circuit is zero".

Mathematically,

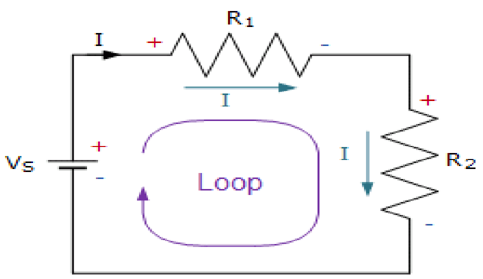
$$\Sigma V + \Sigma IR = 0$$

Sign convention for emf:

- From –ve to +ve terminal of battery ,there is a rise of potential .so emf is taken as +ve.
- From +ve to – ve terminal of battery ,there is a fall of potential so emf is taken as –ve.

Sign convention of voltage drop:

- Along the direction of current in a closed circuit potential drop across the resistor is taken as –ve.
- Against the direction of current, voltage is taken as +ve



Consider the closed path ABCD in above figure .As we travel around the circuit in the clock wise direction different voltage drops will have the following signs.

Applying KVL on above circuit ,

we get $+V_S - IR_1 - IR_2=0$

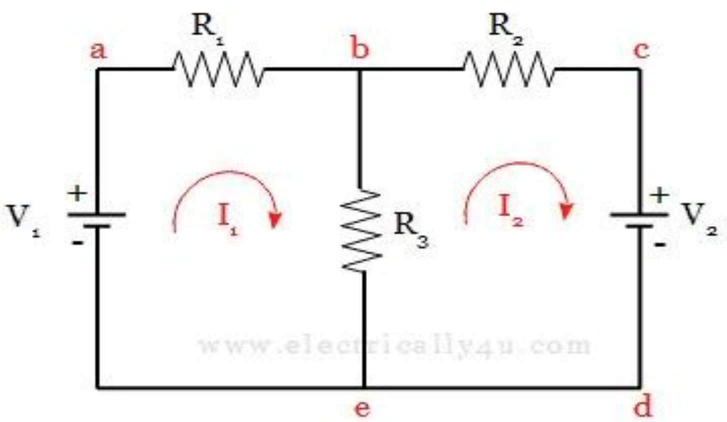
3.2 Mesh Analysis, Mesh Equations by inspection

Mesh Analysis:

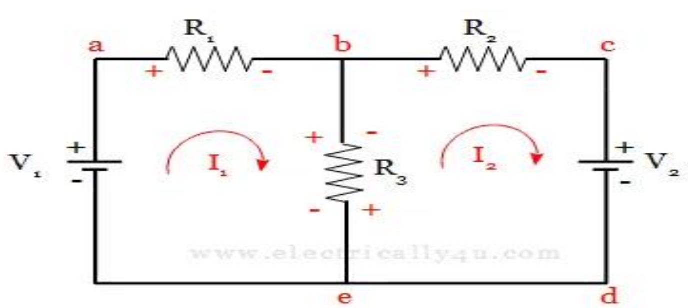
- Mesh analysis is based on Kirchhoff's Voltage Law (KVL).
- In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as Mesh-current method.
- A branch is a path that joins two nodes and it contains a circuit element. If a branch belongs to only one mesh, then the branch current will be equal to mesh current.
- If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

Procedure of Mesh Analysis:

- Consider a simple circuit as shown in the below figure.
- Make sure that the circuit considered for analysis has only voltage sources. If there is any current source in the circuit, use the source transformation method and convert it into a voltage source.
- Label the nodes with either numbers or alphabets.
- Assign the mesh currents in each loop, such that all the current directions are in a clockwise direction



- Along the assumed direction of the current, mark the polarities of a voltage drop across each element. While doing, assign the correct polarity of the voltage source.

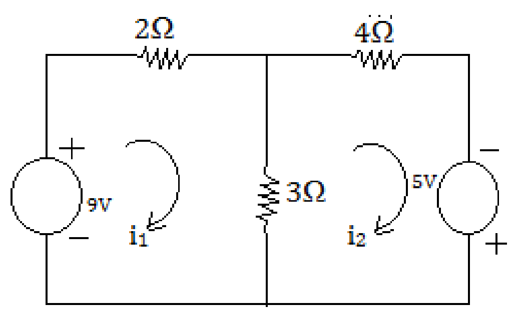


- For each loop, write the current equation by applying KVL to each loop.
- If two currents are flowing in the same branch, it is called a shared branch. In this circuit, mesh currents I_1 and I_2 are flowing through R_3(Branch '*be* ').
- Now, just look at the circuit. For loop1 (*abea*), the current direction is assumed to be from '*b to e* ', the current will be $I_1 – I_2$. But for loop2 (*bcdeb*), the current direction is assumed to be from 'e' *to b* so the current will be $I_2 – I_1$.

- For the above circuit, apply KVL to loop1 and loop2 to get two equations
  $I_1R_1 + (I_1 - I_2)R_3 = V_1 \quad and \quad I_2R_2 + (I_1 - I_2)R_3 = V_1$

Example:

Find the mesh current i_1 and i_2 for the circuit shown in figure



Applying KVL to the mesh 1

$2i_1 + 3(i_1 - i_2) = 9 - - - - -eq1$

Applying KVL to the mesh 2

$4i_2 + 3(i_2 - i_1) = 5 - - - - -eq2$

Solving equation1 & eq2 we get

$i_1 = 3A, \qquad i_2 = 2A$

3 .3 Super mesh Analysis:

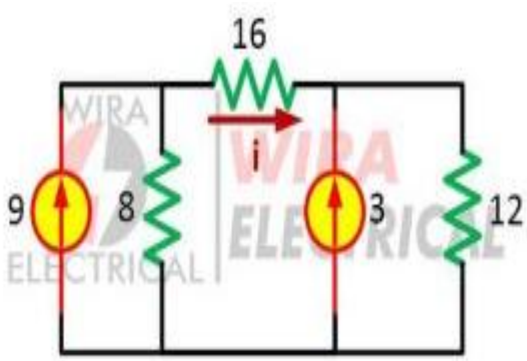
- In mesh analysis, when a current source is present between two mesh, a super mesh analysis has to be performed.

Procedure of Super mesh Current Analysis:

- Redraw the circuit if we can simplify it.
- Make meshes in every loop you can find and assign the labels. It is easier to draw the meshes in the clockwise direction.
- Form a super mesh circuit if you find a current source between two meshes.
- Use KVL and maybe some KCL to the super mesh branch. One KCL is needed for each super mesh branch.
- Solve all the math equations including the super mesh equation.

Examples:

1. Find the value of **i** with supermesh analysis



Solution:

Circuit above is a super mesh with current source.

Observe loop I_1:

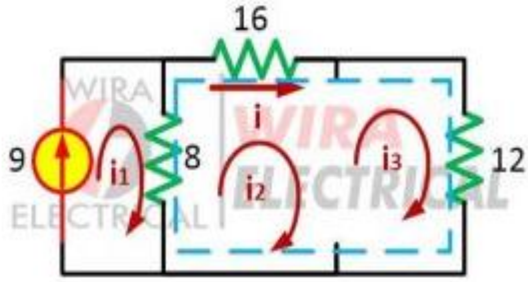
$I_1 = 9\ A$

Observe loop I_2 and I_3:

$I_3 - I_2 = 3\ , A$

$I_3 = 3 + I_2\ , A$ --------------- (1)

Observe super mesh path:



$\sum V = 0$

$8(I_2 - I_1) + 16I_2 + 12I_3 = 0$ --------------- (2)

Substitute Equation.(1) and (2):

$8(I_2 - 9) + 16I_2 + 12(3 + I_2) = 0$

$8I_2 - 72 + 16I_2 + 36 + 12I_2) = 0$

$36I_2 = 36$

$I_2 = \frac{36}{36} = 1, A$

Then $i = I_2 = 1, A$

<u>3.4 Nodal Analysis, Nodal Equations by inspection:</u>

- Nodal analysis is a method that provides a general procedure for analyzing circuits using node voltages as the circuit variables. Nodal Analysis is also called the Node-Voltage Method.
- Nodal Analysis is based on the application of the Kirchhoff's Current Law (KCL).

<u>Types of Nodes in Nodal Analysis:</u>

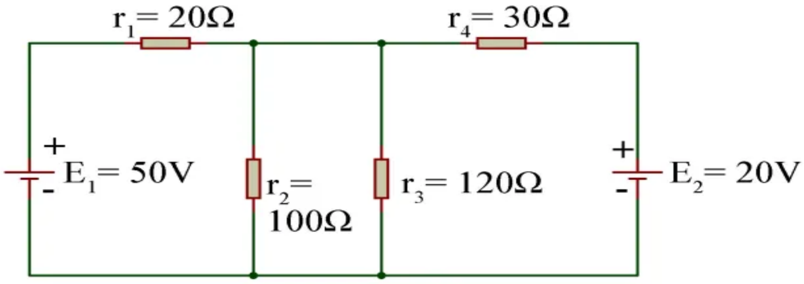
<u>Non-Reference Node –</u> It is a node which has a definite Node Voltage. e.g. Here Node 1 and Node 2 are the Non Reference nodes

<u>Reference Node –</u> It is a node which acts a reference point to all the other node. It is also called the Datum Node.

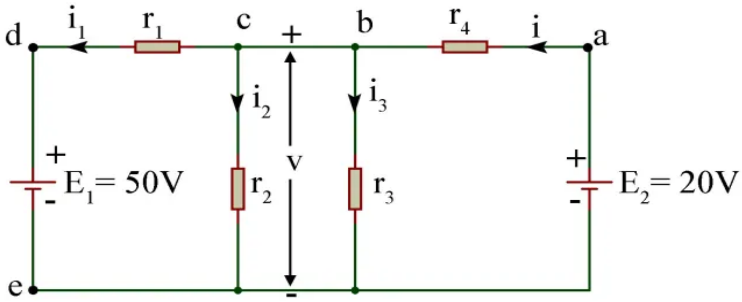
<u>Solving of Circuit Using Nodal Analysis:</u>

- Select a node as the reference node. Assign voltages V_1, V_2… V_{n-1} to the remaining nodes.
- The voltages are referenced with respect to the reference node.
- Apply KCL to each of the non-reference nodes.
- Use Ohm's law to express the branch currents in terms of node voltages.

Q.1 Using Nodal method, find the current through resistor r_2.



Solution: Let us redraw the circuit with naming of the nodes and branch current as shown in figure.



Applying KCL at node "b", (electrically nodes b and c are same)

$i = i_1 + i_2 + i_3$

Assuming the polarity of the voltage v at node c or b, we get

$$\frac{20 - v}{r_4} = \frac{v - 50}{r_1} + \frac{v}{r_2} + \frac{v}{r_3}$$

$$\frac{v - 20}{30} + \frac{v - 50}{20} + \frac{v}{100} + \frac{v}{120} = 0$$

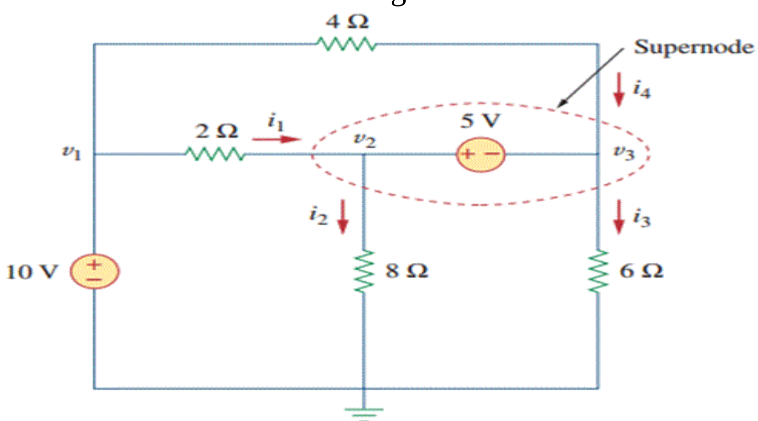
$v = 31.18, V$

$$i_2 = \frac{v}{r_2} = \frac{31.18}{100} = 0.3118, A$$

So current through $r_2 = 311.8, A$

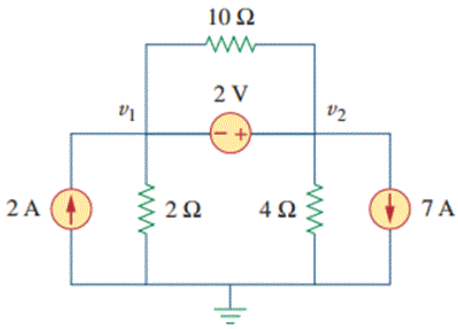
3. 5 Super node Analysis.

- Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes then these two nodes form a generalized node called the Super node.



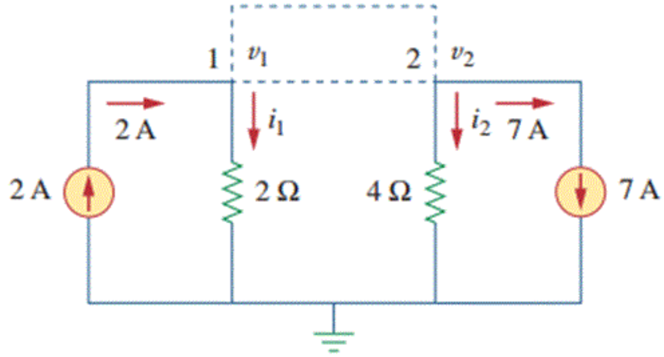
- In the above Figure 5V source is connected between two non reference nodes Node – 2 and Node – 3. So here Node – 2 and Node – 3 form the Super node.

Q.1 Find out the value of V_1 and V_2 of the circuit given below which is containing the super node.



Here 2V voltage source is connected between Node-1 and Node-2 and it forms a Super node with a 10**Ω** resistor in parallel.

Note – Any element connected in parallel with the voltage source forming Super node doesn't make any difference because $V_2 - V_1 = 2V$ always whatever may be the value of resistor. Thus 10 **Ω** can be removed and circuit is redrawn and applying KCL to the super node as shown in figure gives,



Applying KCL at the super node we get

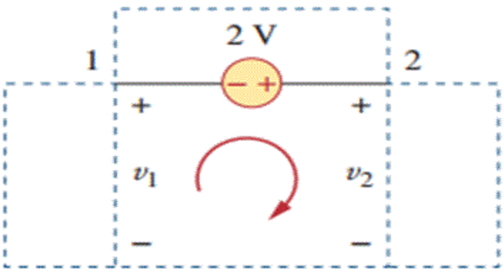
$2 - i_1 - i_2 - 7 = 0 \quad or \;\; i_1 + i_2 + 7 = 2$

or

$$\frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 = 2$$

$2v_1 + v_2 + 28 = 8$

$v_2 = -2v_1 - 20$ ------------(1)



$v_1 + 2 - v_2 = 0$

.or

$v_2 = v_1 + 2$ ------------ (2)

Equating equations 1 &2 we get

$v_1 + 2 = -2v_1 - 20$

Or

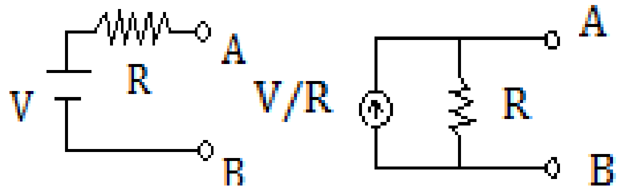
$$3v_1 = -22 \quad or \; v_1 = \frac{-22}{3} = -7.333, V$$

And $v_2 = v_1 + 2 = -7.333 + 2 = 5.333, V$

3. 6 Source Transformation Technique:

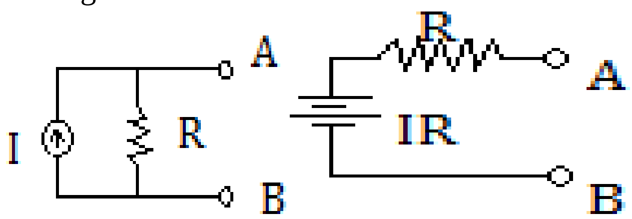
1. voltage source to current source transformation :

- When we converting the voltage source to its equivalent current source then the value of current, $I = \frac{V}{R}$ and the same resistance is connected in parallel with the equivalent current source.
- The equivalent circuit diagram are drawn below.



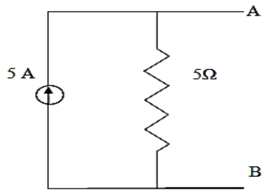
1. Current source to Voltage source transformation :

- When we converting the current source to its equivalent voltage source then the value of voltage, $V = IR$ and the same resistance is connected in series with the equivalent voltage source.
- The equivalent circuit diagram are drawn below.

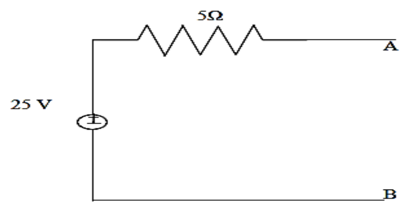


3.7 Solve numerical problems (With Independent Sources Only):

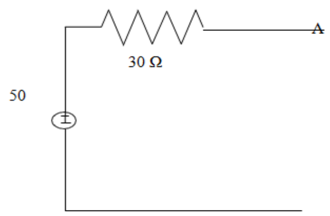
Problem: Determine the equivalent voltage source for the current source shown in fig below



Solution: The voltage across terminals A and B is equal to 25 V.
Since the internal resistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω. And the value of voltage, $V = IR = 5 \times 5 = 25, V$
The equivalent voltage source is shown in fig. below:



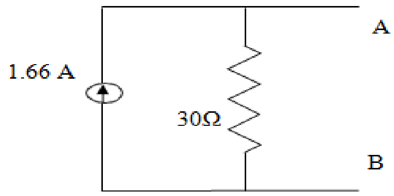
Problem: Determine the equivalent current source for the voltage source shown in fig. below:



Solution : Since the internal resistance for the voltage source is 30Ω, and the value of voltage is 50v

So current, $I = \frac{V}{R} = \frac{50}{30} = 1.66, A$

The equivalent current source is shown in fig. below.



POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

Q-1 What are active and passive elements? [W-17,19]

Ans- A circuit element is said to be active if it has the ability to supply energy that activate a circuit.

Example- Battery, Generator.

A circuit is said to be passive if it absorbs energy

Example- Resistor, Inductor, Capacitor.

Q-2 State Kirchhoff's Laws [W-16,17]

Ans- KCL-Kirchhoff's current law states that the algebraic sum of current entering any node is zero.

KVL-Kirchhoff's voltage law states that the algebraic sum of voltages around any closed path is zero.

Q-3 What is node and mesh? [W-14]

Ans- Node-It is an equipotential point when two or more circuit element connected together.

Mesh- It is a closed path or loop that contains no other loop within it.

Q-4 What do you mean by dependent source? [W-16]

Ans- In dependent sources, the source quantity (voltage, current) depends on a voltage or current existing at some other location in the circuit.

Q-5 What do you mean by linear and non-linear element? Define with suitable example?

Ans-A circuit element is said to be linear if it's V-I characteristics is linear.

Example- Resistor.

A circuit element is said to be non-linear if it does not obey the ohm's law.

Example- Diode, Transistor.

Q-6 What do you mean by super node? [W-2019]

Ans-In a network if a voltage source is common to the two nodes, then that node is called as super node.

POSSIBLE LONG TYPE QUESTIONS

1.State and explain Kirchhoff's Laws.

2.Four lamps are connected in parallel to a 100v supply. Three of the lamp currents are I_{L1}=1.6A, I_{L2}=0.4A. If the total supply current is 5A. Calculate the resistance of each of the four lamps.

CHAPTER NO.- 04

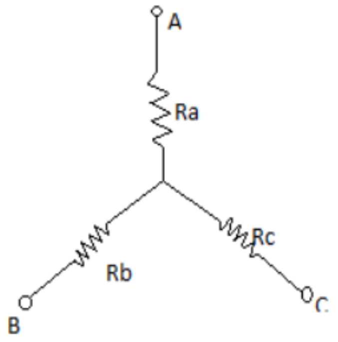
NETWORK THEOREM

Learning Objectives: -

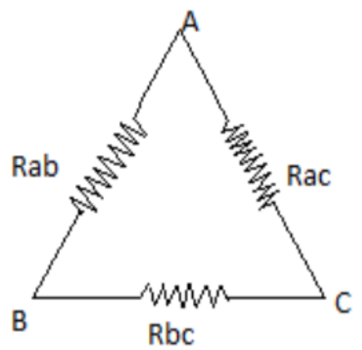
4.1 Star to delta and delta to star transformation
4.2 Super position Theorem
4.3 Thevenin's Theorem
4.4 Norton's Theorem
4.5 Maximum power Transfer Theorem.
4.6 Solve numerical problems (With Independent Sources Only)

4.1 Star to delta and delta to star transformation:

Delta To Star transformation:



Star Connected



Delta Connected

The two systems will be exactly equivalent if the resistance between any pair of terminals A, B and C in figure for the star is the same as that between the corresponding pair for the delta connection when the third terminal is isolated.

For the Y-network resistance between the terminal A and B is

$$R_{ab} = R_a + R_b \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

For the Δ network resistance between the terminals AB is R_{ab}

$$= R_{ab} // (R_{ac} + R_{bc})$$

$$= \frac{R_{ab}(R_{ac}+R_{bc})}{R_{ab}+R_{ac}+R_{bc}} \quad \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$\text{Hence } R_a + R_b = \frac{R_{ab}(R_{ac}+R_{bc})}{R_{ab}+R_{ac}+R_{bc}} \quad \ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

Similarly for Y-network resistance between terminal B and C

$$R_{bc} = R_b + R_c$$

For the Δ network resistance between terminal B and C is R_{bc}

$$= R_{bc} // (R_{ab} + R_{ac})$$

$$= \frac{R_{bc}(R_{ab}+\text{Rac})}{R_{ab}+R_{ac}+R_{bc}}$$

Hence $$R_b + R_c = \frac{R_{bc}(R_{ab}+R_{ac})}{R_{ab}+R_{ac}+R_{bc}} \quad \ldots\ldots\ldots\ldots..(iv)$$

Similarly, we can find Rac between terminal A and C is

$$R_a + R_c = \frac{R_{ac}(R_{ab}+R_{bc})}{R_{ab}+R_{ac}+R_{bc}} \quad \ldots\ldots\ldots\ldots..(v)$$

Subtracting eq.(v) from the sum of eq.(iii) and eq.(iv) yields

$$2R_b = \frac{2(R_{ab}\,R_{bc})}{R_{ab}+R_{ac}+R_{bc}}$$

$$R_b = \frac{(R_{ab}\,R_{bc})}{R_{ab}+R_{ac}+R_{bc}}$$

Subtracting eq.(iv) from the sum of eq.(iii) & eq.(v) yields

$$2R_a = \frac{2(R_{ab}\,R_{ac})}{R_{ab}+R_{ac}+R_{bc}}$$

$$R_a = \frac{(R_{ab}\,R_{ac})}{R_{ab}+R_{ac}+R_{bc}}$$

Similarly subtracting eq.(iii) from the sum of eq.(iv) and eq.(v) yields

$$2R_c = \frac{2(R_{ac}\,R_{bc})}{R_{ab}+R_{ac}+R_{bc}}$$

$$R_c = \frac{2(R_{ac}\,R_{bc})}{R_{ab}+R_{ac}+R_{bc}}$$

Therefore, the equivalent impedance of each arm of the star is given by the product of the impedance of the two delta sides that meet at its ends divided by the sum of their delta impedance.

Star To Delta Conversion:

Similarly, we can find conversion formula for star(Y) to delta (Δ)

$$R_{ab} = \frac{R_a.R_b + R_bR_c + R_c.R_a}{R_c}$$

$$R_{bc} = \frac{R_a.R_b + R_bR_c + R_c.R_a}{R_a}$$

$$R_{ca} = \frac{R_a.R_b + R_bR_c + R_c.R_a}{R_b}$$

4.2 Super position Theorem: -

It states that, ‘In a linear bilateral network containing resistances and energy sources the current flowing through any branch is equal to the algebraic sum of the current produced by each energy source acting separately with all other energy sources set equal to zero’.

Procedure to solve the circuit using superposition theorem:

- Select only one source and replace all other sources with their internal resistance.
- If the source is an ideal currentsource replace it by open circuit.
- If the source is an ideal voltage source, replace it by short circuit.
- Find the current and its direction through the desiredbranch.
- Add all the branch currents to obtain the actual branchcurrent.

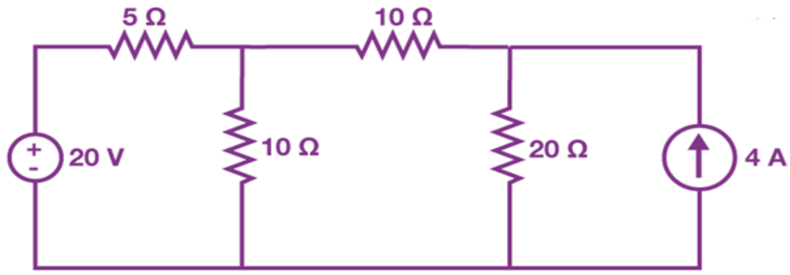
Limitation of Super-position Theorem

- The theorem does not apply to non-linear circuits. The requisite of linearity indicates that the superposition theorem is only applicable to determine voltage and current but not power. Power dissipation is a nonlinear function that does not algebraically add to an accurate total when only one source is considered at a time.

➢ The application of the superposition theorem requires two or more sources in the circuit.

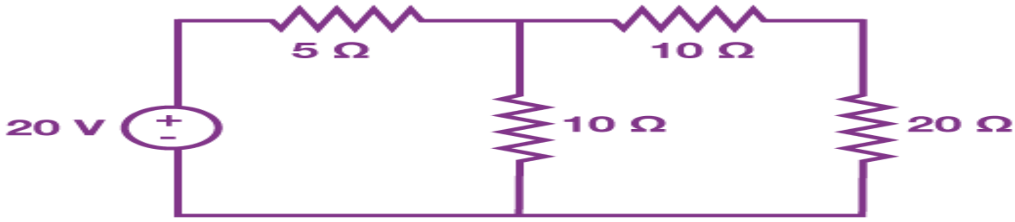
4.6 Numerical problem:

Problem 1: Find the current flowing through 20 Ω using the superposition theorem.



Solution:

Step 1: First, let us find the current flowing through a circuit by considering only the 20 V voltage source. The current source can be open-circuited, hence, the modified circuit diagram is shown in the following figure.



Step 2: The nodal voltage V_1 can be determined using the nodal analysis method.

The nodal equation at node 1 is written as follows:

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1}{10 + 20} = 0$$

$$\Rightarrow \frac{6V_1 - 120 + 3V_1 + V_1}{30} = 0$$

$$\Rightarrow 10V_1 = 120\ OR\ V_1 = 12, V$$

The current flowing through the 20 Ω resistor can be found using the following equation:

$$I_1 = \frac{V_1}{10 + 20}$$

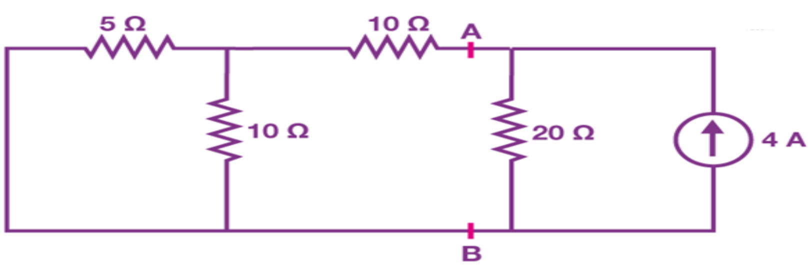
Substituting the value of the V_1 in the above equation, we get

$$I_1 = 0.4, A$$

Therefore, the current flowing through the 20 Ω resistor to due 20 V voltage source is 0.4 A.

Step 3:

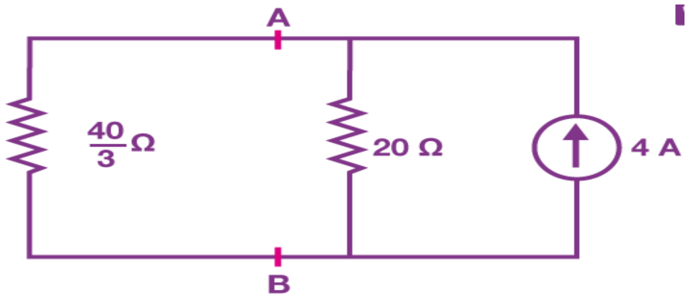
Now let us find out the current flowing through the 20 Ω resistor considering only the 4 A current source. We eliminate the 20 V voltage source by short-circuiting it. The modified circuit, therefore, is given as follows:



In the above circuit, the resistors 5 Ω and 10 Ω are parallel to each other, and this parallel combination of resistors is in series with the 10 Ω resistor. Therefore, the equivalent resistance will be:

$$R_{AB} = \frac{5 \times 10}{5 + 10} + 10 = \frac{40}{3}, Ohm$$

Now, the simplified circuit is shown as follows:



The current flowing through the 20 Ω resistor can be determined using the current division principle.

$$I_2 = I_S \times \frac{R_1}{R_1 + R_2}$$

Substituting the values, we get

$$I_2 = 1.6, A$$

Therefore, the current flowing through the circuit when only 4 A current source is 1.6 A.

Step 4:

The summation of currents I_1 and I_2 will give us the current flowing through the 20 Ω resistor. Mathematically, this is represented as follows:

$$I = I_1 + I_2$$

Substituting the values of I_1 and I_2 in the above equation, we get

$$I = 0.4 + 1.6 = 2, A$$

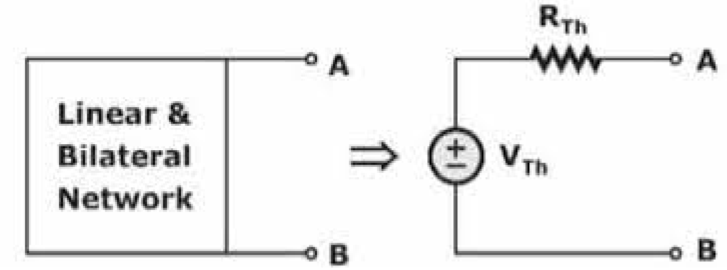
Therefore, the current flowing through the resistor is 2 A.

4.3 Thevenin's Theorem:

Statement:

It states that, "In a linear bi lateral network containing resistances and energy sources, it can be converted in to a single circuit which will contain one equivalent voltage source called as thevenin's voltage(V_{th}) in series with an equivalent resistance called as thevenin's resistance(R_{th}).

The load current can be calculated by $I_L = \frac{V_{th}}{R_{th}+R_L}$

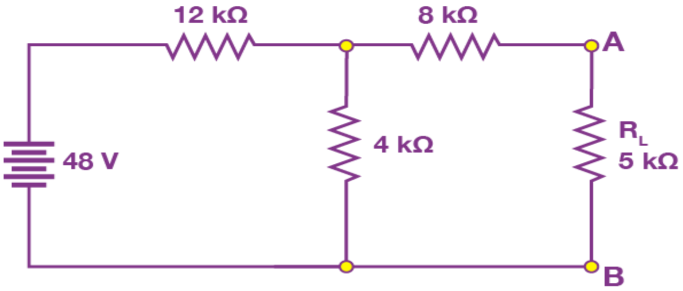


Procedure:

- Let us observe in which element current or voltage drop is to be found out.
- Now open circuit the two terminals of the observed element and it named as A and B.
- For finding Thevenin's resistance(R_{th}), Let us find the equivalent resistance between the point A & B by short circuiting all the voltage sources and open circuiting all the current sources.
- For finding Thevenin's voltage(V_{th}), now connect all the energy sources and apply KVL from point B to point A through the shortest path and it gives open circuit voltage or thevenin's voltage.
- Now draw the thevenin's equivalent circuit and find out the required parameters.

4.6 Numerical problem:

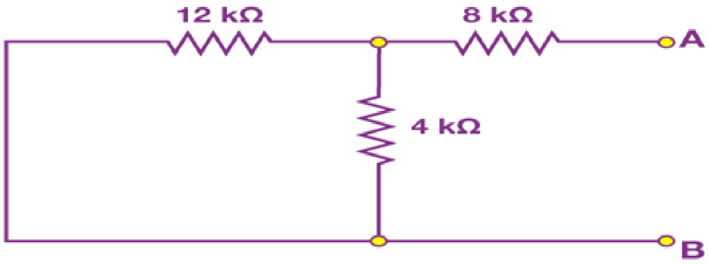
Problem-2: Find out the current and voltage in the load resistance(R_L) of the network given below by applying thevenin's theorem.



Solution :

Step-1(for finding thevenin's resistance R_{th}):

Removing the load resistance(10 ohm) and short circuiting the 24V voltage source, then the circuit will be redrawn below:

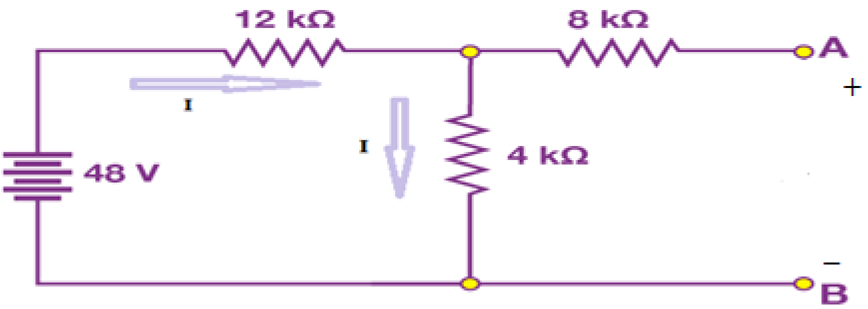


We notice that the 8 kΩ resistor is in series with the parallel connection of 12 kΩ and 4 kΩ resistors. Therefore, the equivalent resistance or the Thevenin's resistance is calculated as follows:

$$R_{AB}\ or\ R_{Th} = 8 + (4//12)$$
$$= 8 + \frac{4 \times 12}{4 + 12}$$
$$= 8 + 3$$
$$\Rightarrow R_{Th} = 11, K\Omega$$

Step-2 (for finding thevenin's voltage V_{th}):

Removing the load resistance R_L and it named as A and B, then the circuit can be redrawn below



Applying KVL to the above mesh we get,

$48 - 12I - 4I = 0$

$\Rightarrow 16I = 48$

$\Rightarrow I = \frac{48}{16} = 3, A$

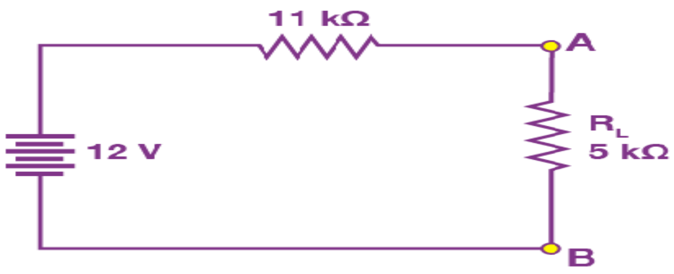
Again applying KVL between point A and B, we get

$V_{AB} - (8 \times 0) - 4I = 0$

$\Rightarrow V_{AB} = 4I = 4 \times 3 = 12, V$

$\Rightarrow V_{AB} = V_{Th} = 12, V$

Now, connect the R_{TH} in series with Voltage Source V_{TH} and the load resistor then thevenin's equivalent circuit is drawn below.



The current flowing through load resistance, $I_L = \frac{V_{th}}{R_{th}+R_L}$

$$= \frac{12}{11+5} = 0.75, mA$$

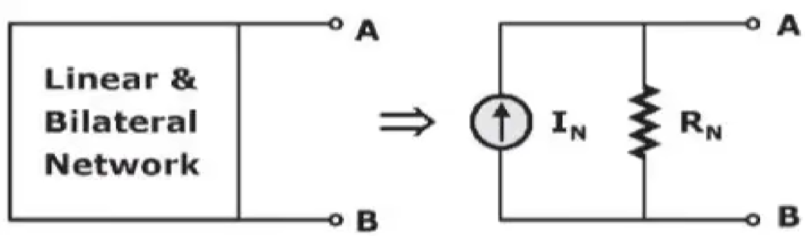
The load voltage is determined as $V_L = 0.75mA \times 5K\Omega = 3.75, V$

4.4 Norton's Theorem:

Statement:

It states that, "In a linear bi lateral network containing resistances and energy sources, it can be converted in to a single circuit which will contain one equivalent current source called as norton's current(I_N) in parallel with an equivalent resistance called as norton's resistance(R_N).

The load current can be calculated by $I_L = I_N \times \frac{R_N}{R_N+R_L}$

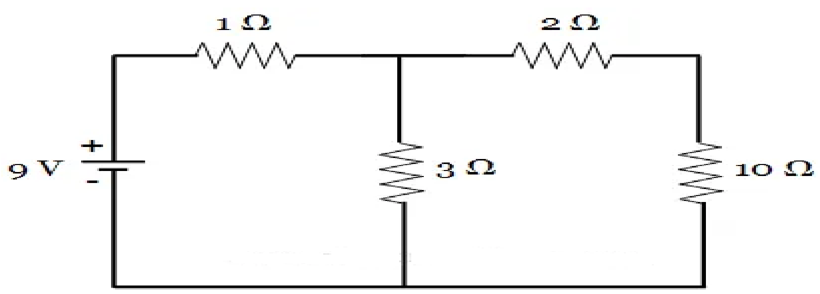


Procedure:

- Let us observe in which element current or voltage drop is to be found out.
- Now open circuit the two terminals of the observed element and it named as A and B.
- For finding norton's resistance(R_N).Let us find the equivalent resistance between the point A & B by short circuiting all the voltage sources and open circuiting all the current sources.
- For finding norton's current(I_N), In the given network short circuit the observe element or load resistance & put the name as A & B. then using any method find out the current in short circuit path between the point A and B.
- Now draw the norton's equivalent circuit and find out the required parameters.

4.6 Numerical problem:

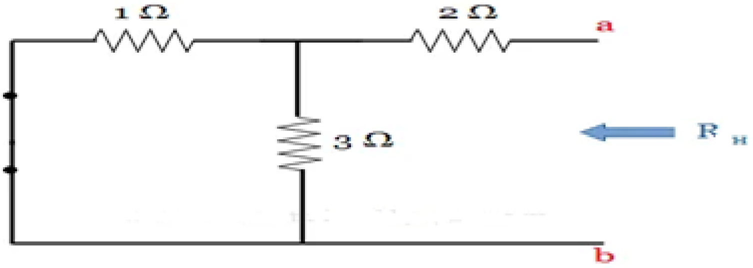
Problem-3: For the given circuit, determine the current flowing through 10 Ω resistor using Norton's theorem.



Solution :

Step-1(for finding Norton's resistance R_N):

Remove the load resistor, short the voltage source and circuit is redrawn as below.



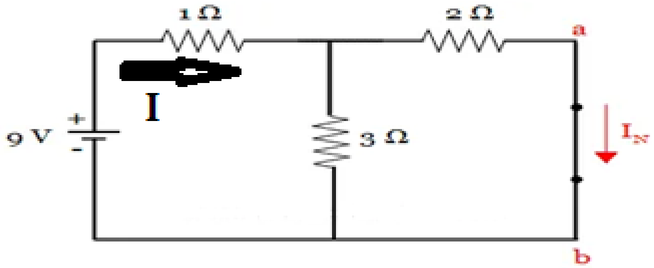
Here 2 Ω resistor is in series with the parallel combination of 1 Ω and 3 Ω resistors. Thus the equivalent value of resistance is obtained as,

$$R_{AB} \; or \; R_N = 2 + (1//3)$$

$$= 2 + \frac{1 \times 3}{1 + 3}$$
$$= 2 + 0.75 = 2.75, \Omega$$
$$R_N = 2.75, \Omega$$

Step-2(for finding Norton's current I_N):

Remove the load resistor(10 Ω), short it and that path current is called as Norton's current, the circuit is redrawn as below.



Here total current, $I = \frac{V}{R_{eq}}$

Here 1 Ω resistor is in series with the parallel combination of 2 Ω and 3 Ω resistors. Thus the equivalent value of resistance is obtained as,

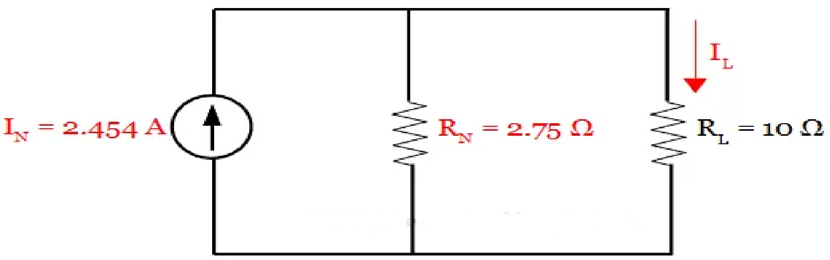
$$R_{eq} = 1 + (2//3)$$
$$= 1 + \frac{2 \times 3}{2 + 3}$$
$$= 1 + 1.2 = 2.2, \Omega$$

So $I = \frac{9}{2.2} = 4.09, A$

By applying current division rule we get,

$$I_N = I \times \frac{3}{3+2} = 4.09 \times \frac{3}{3+2} = 2.454, A$$

The norton's equivalent circuit can be drawn below:



From this circuit, the current through the load R_L = 10 Ω resistor is obtained using current division rule. It is given by,

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

$$= 2.454 \times \frac{2.75}{2.75 + 10} = 0.529, \mathrm{A}$$

The voltage across load is determined as $V_L = 0.529A \times 10\Omega = 5.29, V$

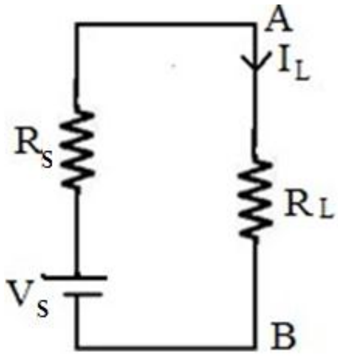
4.5 Maximum power Transfer Theorem.

Statement:

It states that, In a linear bilateral network the maximum power will be transferred from source to load only when the load resistance (R_L) is equal to the source resistance(R_s or R_i).

Proof:

Let's consider the voltage source V_S with its internal resistance R_S is connected to the load resistance R_L



The current in the load resistance, $I_L = \frac{V_S}{R_S+R_L}$

Power delivered to the load resistance is given by

$$P_L = I_L^2 R_L$$

$$= \left(\frac{V_S}{R_S + R_L}\right)^2 R_L$$

$$= \frac{V_S^2 R_L}{(R_S + R_L)^2}$$

The maximum power will be delivered to the load resistance R_L only when

$$\frac{dP_L}{dR_L} = 0$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{dR_L}}\left\{\frac{V_S^2 R_L}{(R_S + R_L)^2}\right\} = 0$$

$$\Rightarrow V_S^2 \frac{d}{dR_L}\left\{\frac{R_L}{(R_S + R_L)^2}\right\}$$

$$\Rightarrow V_S^2 \left[\frac{\frac{dR_L}{dR_L} \times (R_S + R_L)^2 - R_L \frac{d}{dR_L}(R_S + R_L)^2}{(R_S + R_L)^4}\right]$$

$$\Rightarrow V_S^2\left[\frac{(R_S+R_L)^2 - R_L \times 2(R_S+R_L)}{(R_S+R_L)^4}\right]$$

$$\Rightarrow (R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$\Rightarrow (R_S+R_L)^2 = 2R_L(R_S+R_L)$$

$$\Rightarrow (R_S+R_L) = 2R_L$$

$$\Rightarrow R_S = 2R_L - R_L$$

$$\Rightarrow R_S = R_L$$

In this condition the maximum power will transfer from source to load and the maximum power will be

$$P_{max} = I^2 R_L$$

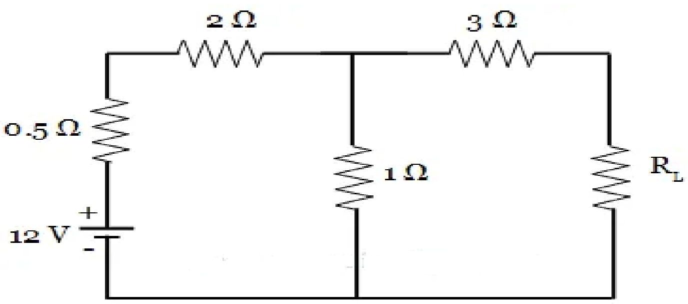
$$= \left(\frac{V_{Th}^2 R_L}{(R_S+R_L)^2}\right)$$

$$= \left(\frac{V_{Th}^2 R_L}{(R_L+R_L)^2}\right) = \left(\frac{V_{Th}^2 R_L}{4{R_L}^2}\right)$$

$$\Rightarrow P_{max} = \left(\frac{V_{Th}^2}{4R_L}\right) \quad \text{Since } R_S = R_L$$

4.6 Numerical problem:

Problem-4: Find the value of R_L at which maximum power is transferred to the load in the following circuit. Also, find the maximum power transferred.



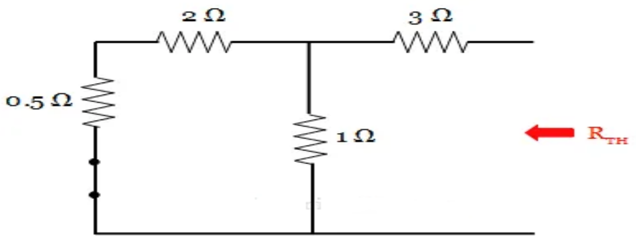
Solution :

We know that the condition for maximum power transfer is, $R_S \text{ or } R_{Th} = R_L$.

Step-1(for finding source resistance R_{th} or R_S):

Thus to find the value of R_L, we need to find the value of Thevenin's resistance R_{Th}.

Short-circuit the voltage source and remove the load resistor then the circuit becomes,



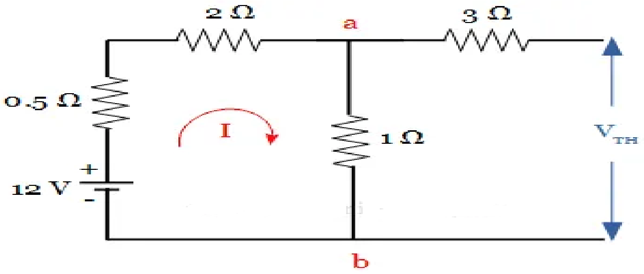
Here 1 Ω resistor and (2 Ω +0.5 Ω) resistors are in parallel with each other. This parallel combination is in series with 3 Ω resistor. Thus, the equivalent resistance is given by,

$$R_{Th} = \frac{1 \times 2.5}{1 + 2.5} + 3 = 3.714, \Omega$$

So

$$R_{Th} = R_L = 3.714, \Omega$$

To find the maximum power that is transferred to the load, we need to find the Thevenin's voltage. So remove the load resistor in the given circuit and redraw it as below,



Since the load resistor is removed, the circuit remains open. Since no current flows through the 3 Ω resistor, , the Thevenin's voltage(V_{TH}) is equal to the voltage across a and b(*i.e.*, V_{ab}).
Current in path ab,

$$I = \frac{12}{0.5 + 2 + 1} = \frac{12}{3.5} = 3.428, A$$

Now, the voltage across 1 Ω resistor(or Thevenin's voltage V_{TH}) is given by,

$$V_{ab} = V_{Th} = I \times 1\Omega = 3.428 \times 1 = 3.428, V$$

Therefore, Maximum Power delivered to the load is given by,

$$P_{max} = \left(\frac{V_{Th}^2}{4R_L}\right) = \frac{(3.428)^2}{4 \times 3.714} = 0.7912, \mathrm{W}$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

Q.1 State super position theorem. [W-19,20]
Ans: It states that in a linear bilateral network containing two or more energy sources, the current flowing through any branch is equal to the algebraic sum of current produced by each energy sources acting separately with all the other energy sources set equal to zero.

Q.2 State Thevenin's theorem. [W-14]
Ans: it states that a linear network containing resistances and energy sources can be transferred into single network which will contain one voltage sources called as Thevenin's voltage ($v_{th)}$ in series with an equivalent resistance called as Thevenin's resistance ($R_{th).}$

Q.3 state maximum power transfer theorem [W-09,10,12,14]
Ans: "It states that the maximum power will be transferred or deliver to a load only when the load resistance is equal to the source resistance".

Q.4 State Blondel's theorem[W-19]
Ans: The Blondel's theorem states that the power provided to a system of N conductors is equal to the algebraic sum of the power measured by' N' wattmeter.

Q.5 What do you mean by nodal analysis of AC network? [W-20]
Ans: In the frequency domain network having n-principal nodes, one of them is designated as the reference node and we require (n-1) node voltage equations to solve for desired result. Regarding sign convention of nodal current, we take the current entering the node as -ve while the current leaving the node is +ve.

Q.6 What do you mean by linear and non-linear element? Define suitable examples? [S-18]

Ans: **<u>linear element:</u>**

A linear element is an electrical element with a linear relationship between current and voltage

Ex-resistors, inductor & capacitor.

<u>Non-linear element:</u>

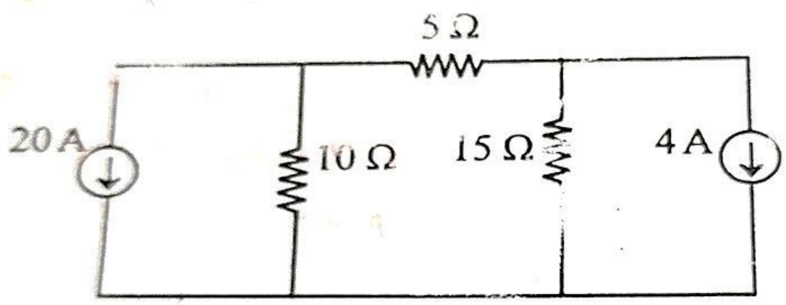
In an electric circuit, a nonlinear element or nonlinear device is an electrical element which does not have a linear relationship between current and voltage.

Ex-diode, transistor & Zener diode.

POSSIBLE LONG TYPE QUESTIONS

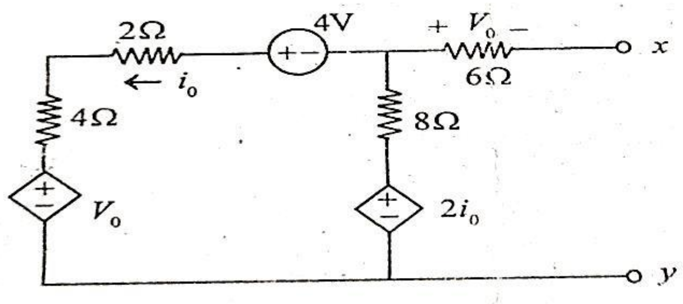
Q.1 State and explain maximum power transfer theorem. [W-16,18,19]

Q.2 Using superposition theorem find the current flowing in 10Ω resistor[W-16,18]

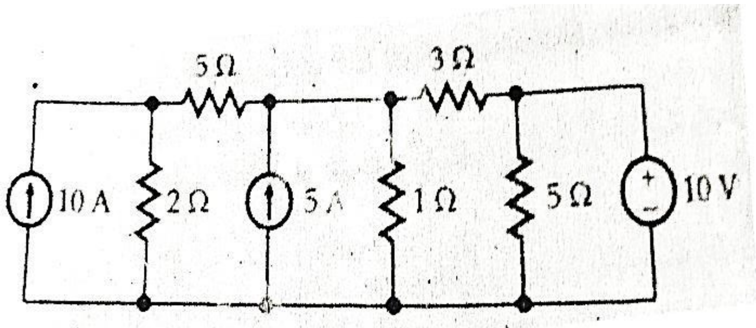


Q.3 Describe briefly about π section of a network? [W-20]

Q.4 Find Norton's equivalent for the network at the left of terminal xy[S-19]

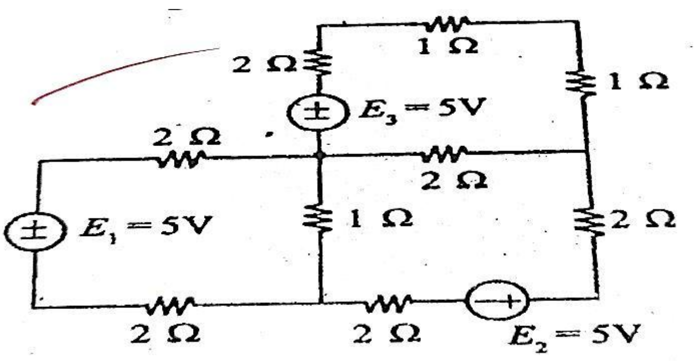


Q.5 Find the current in the 3Ω resistor for the ckt. shown in figure by using Thevenin theorem.



Q.6 Explain star-delta transformation? [S-18]

Q.7 Using nodal method, find the battery current in the given circuit.[W-18]



CHAPTER NO.: 01
MAGNETIC CIRCUIT

Learning Objectives:

1 .1 Introduction
1 .2 Magnetizing force, Intensity, MMF, flux and their relations
1 .3 Permeability, reluctance and permeance
1. 4 Analogy between electric and Magnetic Circuits
1. 5 B-H Curve,
1 .6 Series & parallel magnetic circuit.
1.7 Hysteresis loop.

1.1 Introduction:

- A magnetic circuit is defined as a closed path followed by the magnetic flux.
- A magnetic circuit consists of a core of materials having high permeability like iron, soft steel etc. It is because these materials offer very small opposition to the flow of magnetic flux.

1. 2 Magnetizing force, Intensity, MMF, flux and their relations:

Magnetic flux(φ):

- It is defined as the imaginary lines which are extended from north pole and terminates at south pole.
- It is denoted as φ and its unit is weber (Wb)

Magneto motive Force (M.M.F):

- It is defined as the amount of ampere turns it links.
- It is also defined as the product of magnetic flux and reluctance.
- It drives the flux through a magnetic circuit.
- The unit of M.M.F. is ampere–turn (AT)

Magnetic Field Intensity (H) / Magnetizing force:

- It is defined as the amount of force experienced by a unit north pole placed in a magnetic field.

$$\text{Magnetic field intensity(H)} = \frac{\text{Magnetomotive force(F)}}{\text{Mean length of the magnetic path(L)}}$$

- Its unit is N/m.
- It is also defined as the ratio between flux density and permeability of a medium.

$$i.e\ H = \frac{B}{\mu}, \frac{AT}{m}$$

Flux density(B):

- It is the amount of fluxes passing through the unit area.

$$\text{flux density(B)} = \frac{\varphi}{A}, \frac{Wb}{m^2}$$

1 .3 Permeability, reluctance and permeance:

Permeability(μ):

- It is defined as the ratio between the magnetic flux density and the magnetic field intensity.

$$i.e\ permeability(\mu) = \frac{magnetic\ flux\ density(B)}{Magnetic\ field\ intensity(H)} = \frac{B}{H}$$

- It is also defined as the product of absolute permeability(μ_0) and the relative permeability(μ_r).

$$i.e\ permeability(\mu) = absolute\ permeability(\mu_0) \times relative\ permeability(\mu_r)$$

- Its unit is hennery/meter(H/m).

Where $\mu_0 = 4\pi \times 10^{-7}, \frac{H}{m}$ $\ and\ \mu_r = 1\ for\ air$

Reluctance(S):

- It is defined as the property of a material which opposes the magnetic flux.
- It is denoted as S and its unit is AT/Wb.

$$i.e\ reluctance(S) = \frac{mmf}{magnetic\ flux}, \frac{Ampere\ turn(AT)}{weber(wb)}$$

Permeance(P):

- It is the reciprocal of reluctance.

$$i.e\ Permeance(P) = \frac{1}{Reluctance} = \frac{1}{S}, \frac{weber(wb)}{Ampere\ turn(AT)}$$

1. 4 Analogy between electric and Magnetic Circuits:

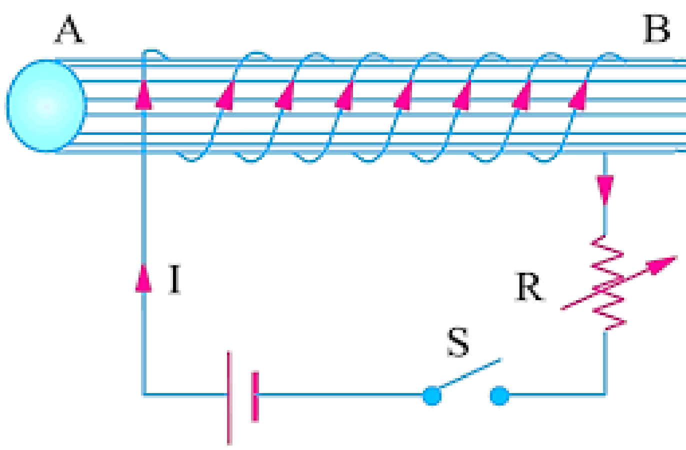
Electric circuit	Magnetic Circuit
EMF drives the electric current.	M.M.F drives the magnetic flux
Resistance opposes the flow of current.	Reluctance opposes the flow of magnetic flux
In electric circuit current flows.	In magnetic circuit flux flows.
Conductance is the reciprocal of resistance.	Permeance is the reciprocal of reluctance.

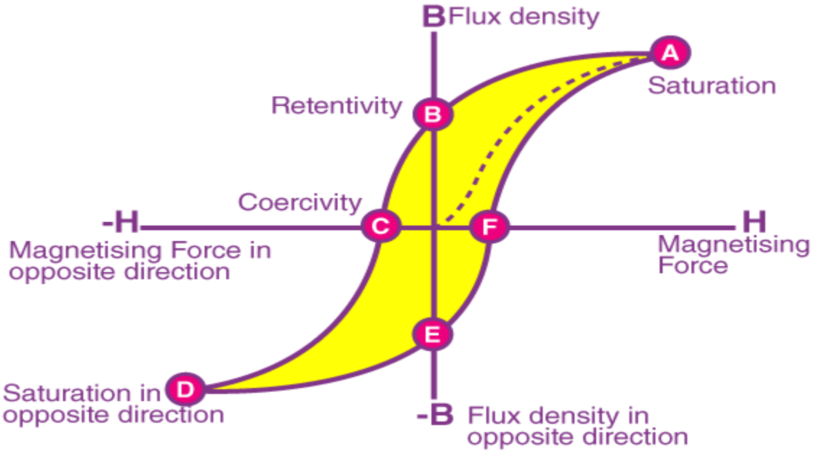
1. 5 B-H Curve:

- The B-H curve is generally used to describe the nonlinear behaviour of magnetization that a ferromagnetic material obtains in response to an applied magnetic field.

1.7 Hysteresis loop:

- Magnetic hysteresis is the property of a magnetic material in which flux density(B) lags behind the magnetic field intensity(H).
- If flux density is taken along y-axis and field intensity along x-axis with different values a graph such plotted is called as hysteresis loop.





- The magnetic flux density (B) is increased when the magnetic field strength(H) is increased from 0 (zero).
- With increasing the magnetic field there is an increase in the value of magnetism and finally reaches point A which is called saturation point where B is constant.
- With a decrease in the value of the magnetic field, there is a decrease in the value of magnetism. But at B and H are equal to zero, substance or material retains some amount of magnetism is called retentivity or residual magnetism.
- When there is a decrease in the magnetic field towards the negative side, magnetism also decreases. At point C the substance is completely demagnetized.
- The force required to remove the retentivity of the material is known as Coercive force (C).
- In the opposite direction, the cycle is continued where the saturation point is D, retentivity point is E and coercive force is F.
- Due to the forward and opposite direction process, the cycle is complete and this cycle is called the hysteresis loop.

Retentivity:

- The amount of magnetization present when the external magnetizing field is removed is known as retentivity.
- It is a material's ability to retain a certain amount of magnetic property while an external magnetizing field is removed.
- The value of B at point b in the hysteresis loop.

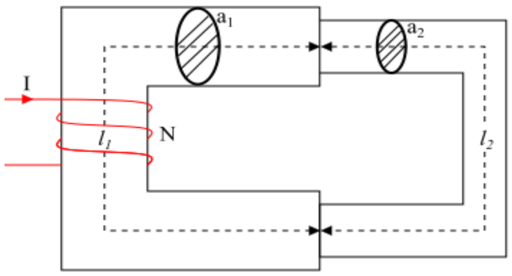
Coercivity:

- The amount of reverse(-ve H) external magnetizing field required to completely demagnetize the substance is known as coercivity of substance.
- The value of H at point c in the hysteresis loop.

1. 6 Series & Parallel magnetic circuit:

Series magnetic circuit:

- When the same magnetic flux φ flows through each part of the magnetic circuit, then the circuit is called as series magnetic circuit.
- Consider a composite series magnetic circuit (a series magnetic circuit that has parts of different dimensions and materials is called a composite series magnetic circuit) consisting of two different magnetic materials of different relative permeability.
- Each part of this series magnetic circuit will offer reluctance to the magnetic flux φ. Since the different parts of the magnetic circuit are in series, the total reluctance is equal to the sum of reluctances of individual parts.

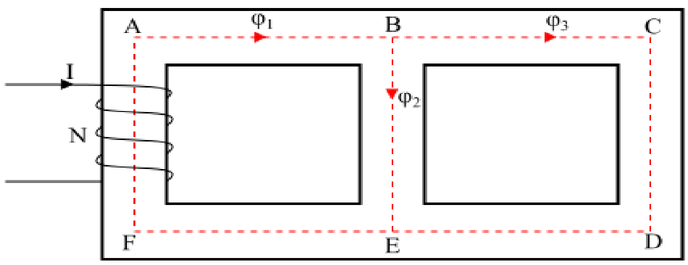


Total reluctance, $S_T = S_1 + S_2$

$$S_T = \frac{L_1}{\mu_0 \mu_{r1} \alpha_1} + \frac{L_2}{\mu_0 \mu_{r2} \alpha_2}$$

Parallel magnetic circuit:

- A magnetic circuit having two or more than two paths for the magnetic flux is called a parallel magnetic circuit.
- Consider a coil of N turns wound on limb AF carries an electric current of I amperes.
- The magnetic flux φ_1 set up by the coil divides at B into two parts i.e the magnetic flux φ_2 passes along the path BE and the magnetic flux φ_3 passes along the path BCDE
- Therefore the total flux, $\varphi_1 = \varphi_2 + \varphi_3$



POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

Q.1. Define flux density .[S-11]

Ans- it is given by the flux passing per unit area through a plane at right angle .

Q.2-Define Permeability.[S-19]

Ans- Permeability of a material (μ) is its conducting power for magnetic lines of force. It is the ratio of the flux density. (B) Produced in a material to the magnetic field strength (H) i.e., μ = B/H

Q.3- Define Reluctance.[W-20,18]

Ans- Reluctance(s) is a measure of the opposition offered by a magnetic circuit to the setting up of the flux. Reluctance is the ratio of magneto motive force to the flux. Thus

S=MMF/ flux (φ)

POSSIBLE LONG TYPE QUESTIONS

Q.1- Explain hysteresis loop with diagram.[W-18,20,15,16]

Q.2- Discuss briefly the B-H curve of ferromagnetic material.[W-17,18]

Q.3- Write down comparison between magnetic & electric circuit. [W-20,S-18]

Q.4- Differentiate between magnetic & electric circuit.

CHAPTER NO.- 02
COUPLED CIRCUITS

Learning Objectives:

2. 1 Self Inductance and Mutual Inductance.
2. 2 Conductivity coupled circuit and mutual impedance
2.3 Dot convention.
2 .4. Coefficient of coupling .
2.5 Series and parallel connection of coupled inductors.
2. 6 Solve numerical problems.

Introduction of coupled circuits:

- It is defined as the interconnected loops of an electric network through the magnetic circuit.
- There are two types of induced emf.

1. Statically Induced emf.
2. Dynamically Induced emf.

1. Statically Induced emf:

- Here the conductors are remain in stationery and flux linked with it changes by increasing or decreasing.

Faraday's Laws of Electro-Magnetic induction:

First Law:

- Faraday's first law of electromagnetic induction states that whenever the flux of magnetic field through the area bounded by a closed loop changes, an emf is produced in the loop.

Second Law:

- It states that the magnitude of induced emf is equal to the rate of change of flux linkages.
- In other word it states that the emf induced is directly proportional to the rate of change of flux and number of turns.

Mathematically:

$$e \propto \frac{d\varphi}{dt}$$

$$e \propto N$$

$$e \propto -N\frac{d\varphi}{dt}$$ ('- ve' sign is due to Lenz's Law)

Where (e = induced emf, N = No. of turns, φ = flux)

Inductance:

- It is defined as the property of the substance which opposes any change in Current & flux.
- Its Unit is Henry

Fleming's Right Hand Rule:

- It states that "hold your right hand with fore-finger, middle finger and thumb at right angles to each other. If the fore-finger represents the direction of field, thumb represents the direction of motion of the conductor, then the middle finger represents the direction of induced emf."

Lenz's Law:

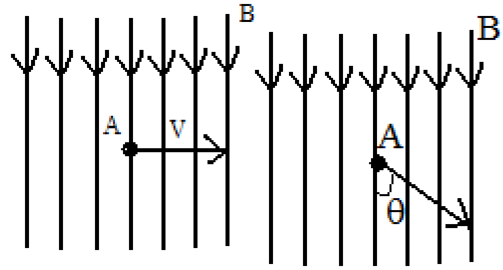
- It states that electromagnetically induced current always flows in such a direction that the action of magnetic field set up by it tends to oppose the vary cause which produces it.

OR

- It states that the direction of the induced current (emf) is such that it opposes the change of magnetic flux

2.Dynamically Induced emf:

- In this case the field is stationary and the conductors are rotating in an uniform magnetic field at flux density 'B" Wb/m^2 and the conductor is lying perpendicular to the magnetic field.



Let 'l' is the length of the conductor and it moves a distance of 'dx' nt in time 'dt' second.

The area(A) swept by the conductor = ldx

Hence the flux cut $= \mathrm{BA}\cos\theta = \mathrm{B1dx} \quad \text{for } \theta = 0$

$\mathrm{E} = \frac{d\varphi}{dt}$ for one turn

$= \frac{\mathrm{dB1dx}}{\mathrm{dt}}$

$\mathrm{E} = \mathrm{B1v}$ (where velocity (V) $= \frac{dx}{dt}$)

If the conductor is making an angle 'θ' with the magnetic field, then

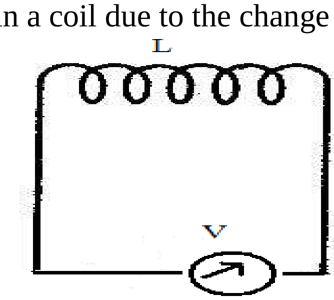
$\mathrm{E} = \mathrm{B1v}\sin\theta$

Statically Induced emf is is divided into two types .

1. Self-induced emf
2. Mutually-induced emf.

Self-induced emf :

- It is defined as the emf induced in a coil due to the change of its own flux linked with the coil.



- If current through the coil is changed then the flux linked with its own turn will also change which will produce an emf is called self-induced emf.

$$E_L = -\frac{d\varphi}{dt} = -L\frac{di}{dt}$$

2.1 Self Inductance and Mutual Inductance:

Self-Inductance:

- It is defined as the property of the coil due to which it opposes any change (increase or decrease) of current or flux through it

Co-efficient of Self-Inductance (L) :

We know , $\varphi \propto I = L \times I$ (for one turn)

$\Rightarrow N\varphi = L \times I$ (for N turns)

$$\Rightarrow L = \frac{N\varphi}{I}$$

Where L = Co-efficient of self-induction,
N = Number of turns
φ = flux , I = Current

- It is defined as the ratio of weber turns per ampere of current in the coil.

OR

- It is the ratio of flux linked per ampere of current in the coil

1st Method for 'L':

$$L = \frac{N\varphi}{I}$$

$$L_1 = \frac{N_1\varphi_1}{I_1} \quad (\text{self} - \text{induced emf in coil } 1)$$

$$L_2 = \frac{N_2\varphi_2}{I_2} \quad (\text{self} - \text{induced emf in coil } 2)$$

2nd Method for L :

We know, $E_L = -\frac{d\varphi}{dt} = -\frac{d(LI)}{dt} = -L\frac{di}{dt}$

$$\Rightarrow L = -\frac{E_L}{\frac{di}{dt}} \quad , (E_L = \text{ self induced emf})$$

3rd Method for L :

We know, $B = \mu_0 nI = \mu_0 \frac{N}{L} I$ (n = no of turn per unit length)

$$\varphi = BA = \mu_0 \frac{N}{L} IA$$

$$L = \frac{N\varphi}{I} = \frac{\mu_0 N^2 A}{1}, (\text{for air cored solenoid coil})$$

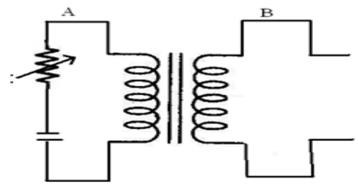
$$L = \frac{\mu_0 \mu_r N^2 A}{1}, (\text{for solenoid coil wound over high permeability material})$$

Mutually Induced emf :

- It is defined as the emf induced in one coil due to change in current in other coil.

- Consider two coils 'A' and 'B' lying close to each other, an emf will be induced in coil 'B' due to change of current in coil 'A' by changing the position of the rheostat.

$$E_M = -\frac{d\varphi}{dt} = -M\frac{di}{dt}$$



Mutual Inductance:

- It is defined as the emf induced in coil 'B' due to change of current in coil 'A' is the ratio of flux linkage in coil 'B' to 1 amp. Of current in coil 'A'. Co-efficient of Mutual Inductance (M)

We know, $\varphi_A \propto I_A$

$\Rightarrow \varphi_A = M\, I_A$ (for one turn)

$\Rightarrow N_B\varphi_A = M\, I_A$ (for'NB' turn in second coil B)

$$\Rightarrow M = \frac{N_B\varphi_A}{I_A}$$

Where M = Co-efficient of self-induction,
N_B = Number of turns in second coil B ,
φ_A = flux due to A coil , I_A = Current in coil A)

Similarly, $M = \frac{N_A\varphi_B}{I_B}$

- Coefficient of mutual inductance between the two coils is defined as the weber-turns in one coil due to one ampere current in the other.

1st Method for 'M' :

$$M = \frac{N_B\varphi_A}{I_A} \text{ or } M = \frac{N_A\varphi_B}{I_B}$$

2nd Method for M :

$$E_M = -\frac{d\varphi_B}{dt} = -M\frac{dI_B}{dt}$$

$$E_M = -\frac{d\varphi_A}{dt} = -M\frac{dI_A}{dt}$$

E_M = mutual induced emf .

$$M = \frac{E_M}{-\frac{dI_A}{dt}}$$

3rd Method for M :

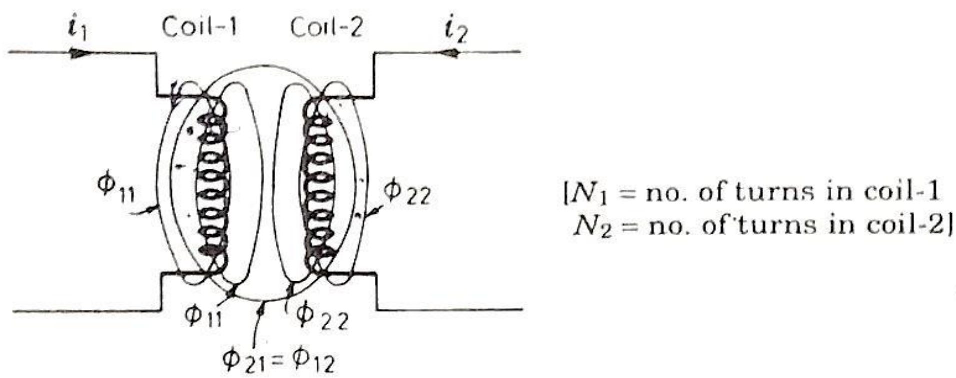
$$M = \frac{N_B\varphi_A}{I_A}$$

$$= \frac{N_B B_A A}{I_A} \qquad (B_A = \mu_0 \frac{N_A}{L} I_A)$$

$$= \frac{\mu_0 N_A N_B A}{L} \quad \text{(for air cored solenoid coil)}$$

$$= \frac{\mu_0 \mu_r N_A N_B A}{L} \quad \text{(for solenoid coil wound over high permeability)}$$

Leakage flux & linkage flux:



Let two coils carry currents I_1 & I_2 .each coil will have leakage flux φ_{11} & φ_{22} as well as mutual flux φ_{12} & φ_{21} where the flux of coil 2 links to coil 1 or flux of coil 1 links to coil 2.

$$\text{So } M = \frac{N_2 \varphi_{12}}{I_1} \text{ or } M = M = \frac{N_1 \varphi_{21}}{I_2}$$

$\varphi_1 = \varphi_{11} + \varphi_{12}$ $\qquad \varphi_1$ = total flux by coil 1

$\varphi_2 = \varphi_{22} + \varphi_{21}$ $\qquad \varphi_2$ = total flux by coil 2

2. 2Conductively coupled circuit and mutual impedance:

In below fig-(a) the loop equations are

$$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$V_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

(M is +ve I_1 & I_2 both enter the coils through dot ends)

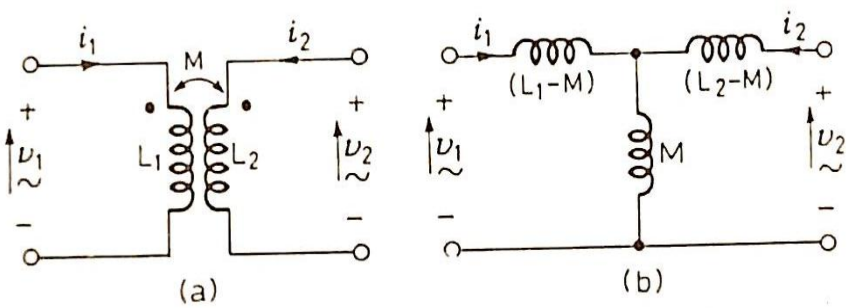


Fig. conductively coupled equivalent circuits

In above fig-(a) the loop equations are

$$V_1 = (L_1 - M)\frac{dI_1}{dt} + M\frac{d(I_1 + I_2)}{dt}$$

$$V_2 = (L_2 - M)\frac{dI_2}{dt} + M\frac{d(I_1 - I_2)}{dt}$$

Simplification becomes

$$V_1 = L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt}$$

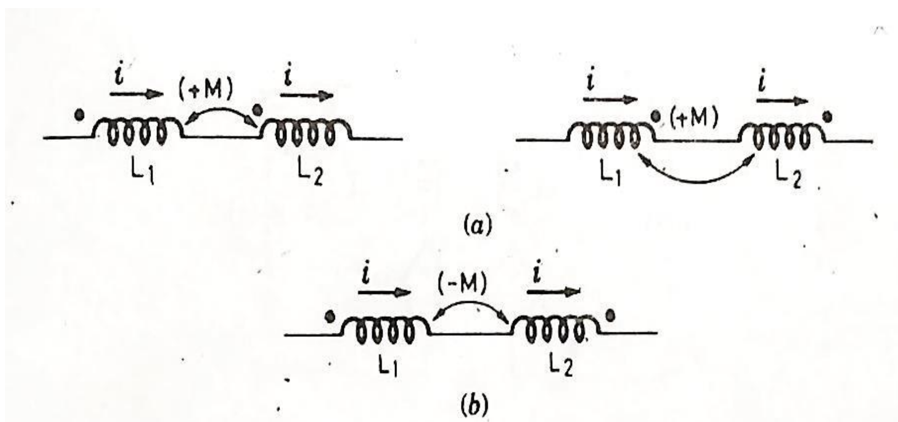
$$V_1 = L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt}$$

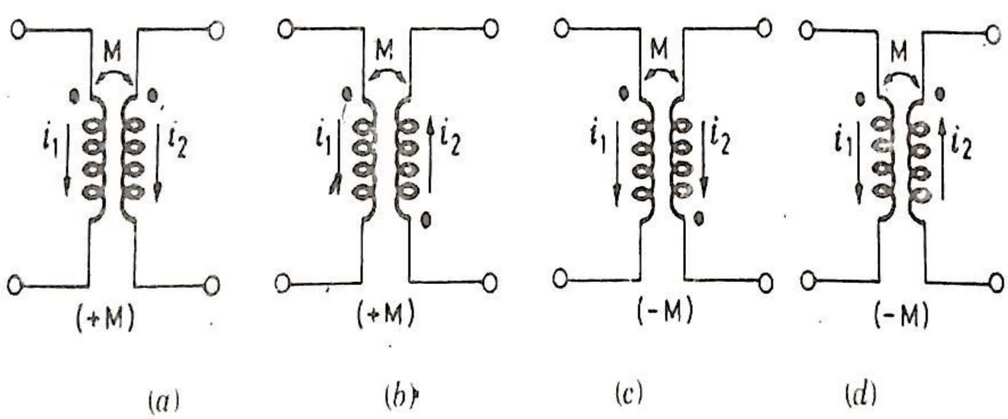
Thus we see that both fig (a),(b) are called conductively equivalent of the magnetic ckt.

We may represent $Z_A = (L_1 - M),\ Z_B = (L_2 - M),\ Z_C = M,$

2. 3 Dot convention:

- To determine the relative polarity of the induced voltage in the coupled coil, the coils are marked with dots.
- When the currents through each mutually coupled coils are going away from the dot or towards dot, the mutual inductance(M) is +ve
- When the current through the coil is leaving the dot for one coil & entering the other, the mutual inductance (M) is –ve.





2.4 Co-efficient of Coupling(K):

It is defined as the fraction of total flux that links the coils.

$$K = \frac{\varphi_{12}}{\varphi_1} = \frac{\varphi_{21}}{\varphi_2}$$

We know $M^2 = \frac{N_2\varphi_{12}}{I_1} \times \frac{N_1\varphi_{21}}{I_2}$

$$= \frac{N_2K\varphi_1}{I_1} \times \frac{N_2K\varphi_2}{I_1}$$

$$= K^2\left(\frac{N_1\varphi_1}{I_1}\right) \times \left(\frac{N_2\varphi_2}{I_2}\right)$$

$$= K^2L_1L_2$$

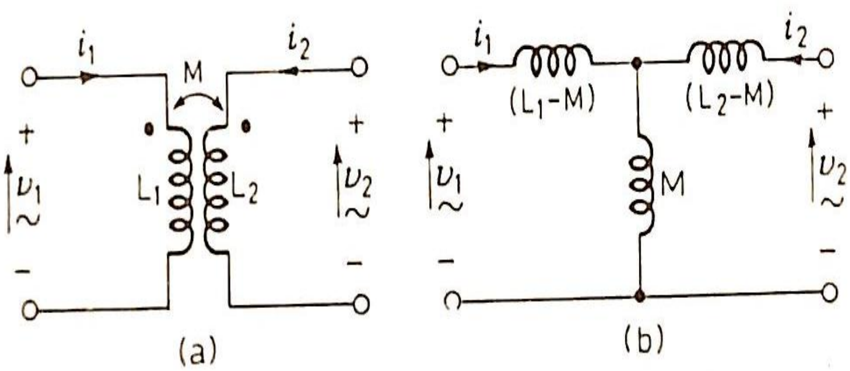
$$\Rightarrow M = K\sqrt{L_1L_2}$$

$$\Rightarrow K = \frac{M}{\sqrt{L_1L_2}}$$

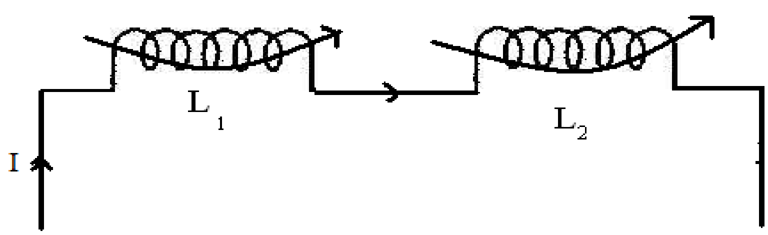
Where ‘K’ is known as the co-efficient of coupling.

- Co-efficient of coupling is defined as the ratio of mutual inductance between two coils to the square root of their self- inductances.

2.5 Series and parallel connection of coupled inductors:



Inductances In Series (Additive) :



Fluxes are in the same durection

Let M = Co-efficient of mutual inductance

L_1 = Co-efficient of self-inductance of first coil

L_2 = Co-efficient of self-inductance of second coil

EMF induced in first coil due to self-induction

$$E_{L1} = -L_1 \frac{di}{dt}$$

Mutually induced emf in first coil

$$E_{M1} = -M \frac{di}{dt}$$

EMF induced in second coil due to self induction

$$E_{L2} = -L_2 \frac{di}{dt}$$

Mutually induced emf in second coil

$$E_{M2} = -M \frac{di}{dt}$$

Total induced emf

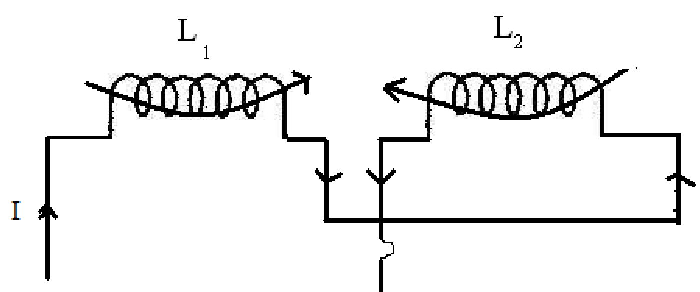
$$E_{net} = E_{L1} + E_{M1} + E_{L2} + E_{M2}$$

$$-L_{eq} \frac{di}{dt} = \left(-L_1 \frac{di}{dt}\right) + \left(-M \frac{di}{dt}\right) + \left(-L_2 \frac{di}{dt}\right) + \left(-M \frac{di}{dt}\right)$$

$$\Rightarrow L_{eq} = L_1 + M + L_2 + M$$

$$\Rightarrow L_{eq} = L_1 + L_2 + 2M$$

Inductances In Series (Subtractive) :



(Fluxes are opposite in direction)

Let M = Co-efficient of mutual inductance
 L_1 = Co-efficient of self-inductance of first coil
 L_2 = Co-efficient of self-inductance of second coil

EMF induced in first coil due to self-induction

$$E_{L1} = -L_1 \frac{di}{dt}$$

Mutually induced emf in first coil

$$E_{M1} = -\left(-M \frac{di}{dt}\right) = M \frac{di}{dt} \qquad \text{(M is – ve)}$$

EMF induced in second coil due to self induction

$$E_{L2} = L_2 \frac{di}{dt}$$

Mutually induced emf in second coil

$$E_{M2} = -\left(-M\frac{di}{dt}\right) = M\frac{di}{dt} \qquad \text{(M is – ve)}$$

Total induced emf

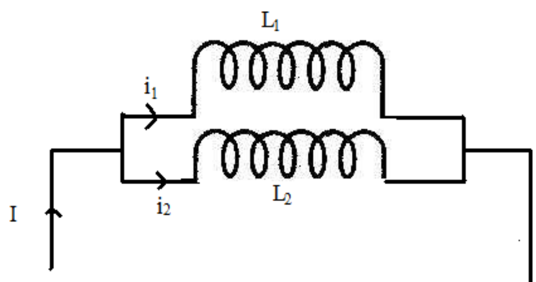
$$E_{net} = E_{L1} + E_{M1} + E_{L2} + E_{M2}$$

$$-L_{eq}\frac{di}{dt} = \left(-L_1\frac{di}{dt}\right) + \left(M\frac{di}{dt}\right) + \left(L_2\frac{di}{dt}\right) + \left(M\frac{di}{dt}\right)$$

$$\Rightarrow L_{eq} = L_1 - M + L_2 - M$$

$$\Rightarrow L_{eq} = L_1 + L_2 - 2M$$

Inductances In Parallel:



Let two inductances of L_1 & L_2 are connected in parallel and their co-efficient of mutual inductance between them is M.

$$I = I_1 + I_2$$

$$\Rightarrow \frac{di}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

We can write

$$E = -L_1\frac{dI_1}{dt} + \left(-M\frac{dI_2}{dt}\right) ------ (1)$$

$$E = -L_2\frac{dI_2}{dt} + \left(-M\frac{dI_1}{dt}\right) ------ (2)$$

M is multiplied both side in eq1 then it becomes

$$ME = -ML_1\frac{dI_1}{dt} + \left(-M^2\frac{dI_2}{dt}\right) ------ (3)$$

L_1 is multiplied both side in eq2 we get

$$L_1E = -L_1L_2\frac{dI_2}{dt} + \left(-L_1M\frac{dI_1}{dt}\right) ------ (4)$$

Substract eq(3) from eq (4) we get

$$E(M - L_1) = \frac{dI_2}{dt}(L_1L_2 - M^2)$$

$$\Rightarrow \frac{dI_2}{dt} = \frac{E(M - L_1)}{(L_1L_2 - M^2)} ------------(5)$$

Similarly, L_2 is multiplied both side in eqn 1

$$L_2E = -L_1L_2\frac{dI_1}{dt} + \left(-L_2M\frac{dI_2}{dt}\right) - - - - - - (6)$$

M is multiplied both side in eq2

$$ME = -ML_2\frac{dI_2}{dt} + \left(-M^2\frac{dI_1}{dt}\right) - - - - - - (7)$$

Substract from eq5 to eq6 we get

$$E(L_2 - M) = \frac{dI_1}{dt}(M^2 - L_1L_2) - - - - - - (8)$$

$$\Rightarrow \frac{dI_1}{dt} = \frac{E(L_2 - M)}{(M^2 - L_1L_2)}$$

$$\Rightarrow \frac{dI_1}{dt} = \frac{-E(L_2 - M)}{-(M^2 - L_1L_2)} = \frac{E(M - L_2)}{(L_1L_2 - M^2)} - - - - - - (9)$$

We know that

$$\frac{di}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} - - - - - - - - - - (10)$$

Putting the value of $\frac{dI_1}{dt}$ and $\frac{dI_2}{dt}$ in equation (10)

$$\frac{di}{dt} = \frac{E(M - L_2)}{(L_1L_2 - M^2)} + \frac{E(M - L_1)}{(L_1L_2 - M^2)}$$

$$\Rightarrow \frac{di}{dt} = E\left[\frac{M - L_2 + M - L_1}{L_1L_2 - M^2}\right]$$

$$\Rightarrow \frac{di}{dt} = E\left[\frac{2M - (L_1 + L_2)}{L_1L_2 - M^2}\right]$$

$$\Rightarrow E = \frac{di}{dt}\left[\frac{L_1L_2 - M^2}{2M - (L_1 + L_2)}\right]$$

$$\Rightarrow -L_{eq}\frac{di}{dt} = \frac{di}{dt}\left[\frac{L_1L_2 - M^2}{2M - (L_1 + L_2)}\right] \qquad \text{Where } E = -L_{eq}\frac{di}{dt}$$

$$\Rightarrow L_{eq} = -\left[\frac{L_1L_2 - M^2}{2M - (L_1 + L_2)}\right]$$

$$\Rightarrow L_{eq} = \left[\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right]$$

2. 6 Solve numerical problems:

Problem:1

Find the total inductance of the three series connected coupled coils. Where the self and mutual inductances are $L_1 = 1H$, $L_2 = 2H$, $L_3 = 5H$, $M_{12} = 0.5H$, $M_{23} = 1H$, $M_{13} = 1H$

Solution:

$L_A = L_1 + M_{12} + M_{13} = 1 + 20.5 + 1 = 2.5, H$

$L_B = L_2 + M_{23} + M_{12} = 2 + 1 + 0.5 = 3.5, H$

$L_C = L_3 + M_{23} + M_{13} = 5 + 1 + 1 = 7, H$

Total inductances

$L_{eq} = L_A + L_B + L_C = 2.5 + 3.5 + 7 = 13, H$ (Ans)

Example 02:

Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500A/s in A induces an emf of 11.25 V in B. Calculate the mutual inductance of the arrangement .If the self-inductance of each coil is 15mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

Solution:

We know that $E_M = -M\frac{dI_A}{dt}$

$$M = \frac{E_M}{-\frac{dI_A}{dt}} = \frac{11.25}{1500} = 7.5 \text{ mH}$$

Now, $L_1 = \frac{N_1\varphi_1}{I_1}$

$$\Rightarrow \frac{\varphi_1}{I_1} = \frac{L_1}{N_1} = \frac{15\times10^{-3}}{750} = 0.02\times10^{-3}, \text{Wb/A}$$

$$K = \frac{M}{\sqrt{L_1L_2}} = \frac{7.5\times10^{-3}}{\sqrt{(15\times10^{-3})^2}} = 0.5$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

Q.1- What do you mean by co-efficient of coupling? [W-18,19,20]

Ans-The co-efficient of coupling between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other coil.

$$K = \frac{M}{\sqrt{L_1L_2}}$$

Q.2-Define Mutual Inductance [W-05]

Ans-mutual inductance may be defined as the ability of one coil (or circuit) to produce an emf in a nearby coil by induction when the current in the first coil changes.

Q-3- State Flemings right hand rule. [W-10]

Ans- It states that "hold your right hand with fore-finger, middle finger and thumb at right angles to each other. If the fore-finger represents the direction of field, thumb represents the direction of motion of the conductor, then the middle finger represents the direction of induced emf."

Q.4- State lens's law [W-18]

Ans- It state that the current induced in a circuit due to a change in flux or a motion in a magnetic field is so directed as to oppose the change in flux & to exert a mechanical force opposing the motion.

Q.5 Find the total inductance-

Ans- Total inductance, $L_{eq} = L_1 + L_2 + 2M$

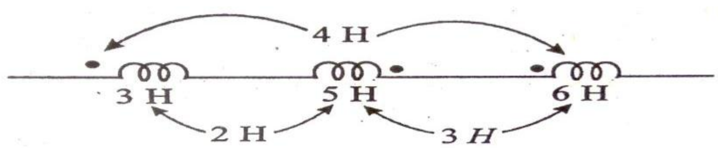
POSSIBLE LONG TYPE QUESTIONS

Q.1- two coils A of 12500 turns & B of 16000 turns lie in parallel planes so that 60% of flux produced in A links coil B. it is found that a current of 5A in A produces flux of 0.6 mwb. Determine

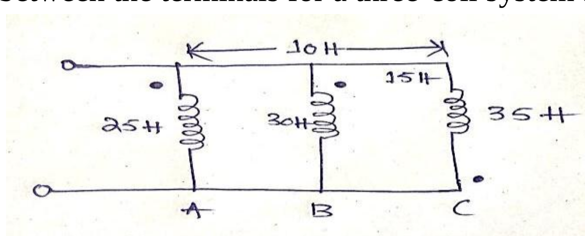
(i) mutual inductance

(ii) coefficient of coupling [W-10]

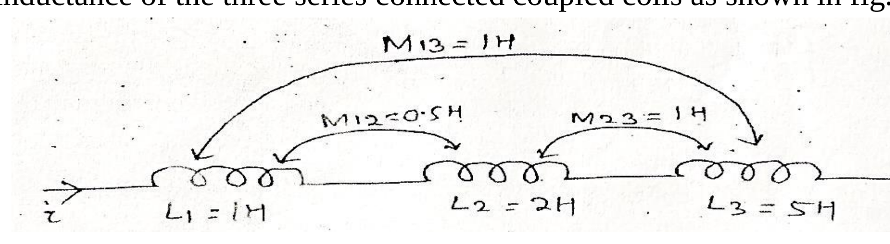
Q.2 Find the total inductance in the circuit shown below.



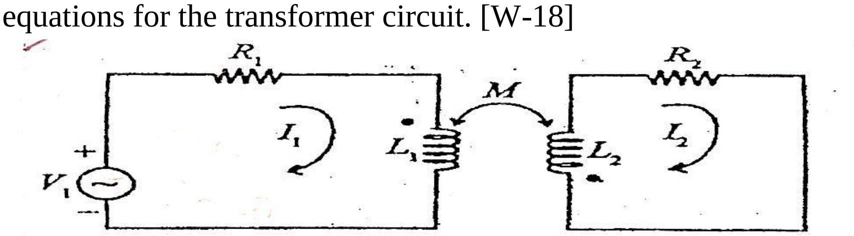
Q.3 Determine the inductance between the terminals for a three-coil system shown below.[W-10]



Q.4 Find the total inductance of the three series connected coupled coils as shown in fig.[W-19]



Q.5 Write the mess equations for the transformer circuit. [W-18]



CHAPTER NO.- 05

AC CIRCUIT AND RESONANCE

Learning objectives:

5.1 A.C. through R-L, R-C & R-L-C Circuit
5.2 Solution of problems of A.C. through R-L, R-C & R-L-C series Circuit by complex algebra method.
5.3 Solution of problems of A.C. through R-L, R-C & R-L-C parallel & Composite Circuits
5.4 Power factor & power triangle.
5.5 Deduce expression for active, reactive, apparent power.
5.6 Derive the resonant frequency of series resonance and parallel resonance circuit
5.7 Define Bandwidth, Selectivity & Q-factor in series circuit.
5.8 Solve numerical problems

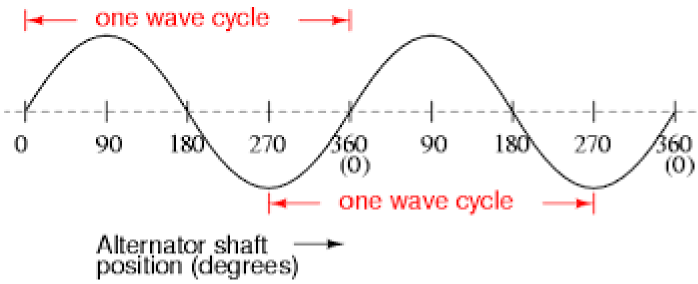
A.C .Fundamentals:

Definition:

Alternating Current (A.C.) is defined as an electric current which changes both its magnitude & direction periodically.

Important Terminology:

1.Cycle: It is defined as one set of + ve & - ve values of an alternating current.



2. Frequency (f): It is defined as the number of wave cycles formed in one second. It is denoted by f & its unit is hertz (Hz).

Mathematically $f = \frac{1}{T}$, Hz

3.Time Period (T) : It is defined as the time taken by an alternating quantity (voltage/current / emf) to complete one cycle. It is denoted by T & its unit is second (S)

4. Amplitude: It is defined as the peak or maximum value of an alternating quantity. It may be +ve or – ve maximum amplitude.

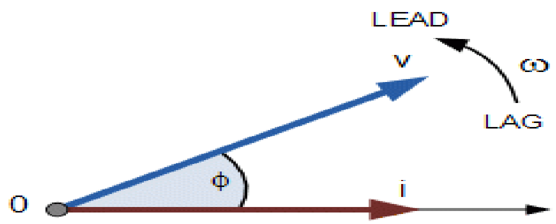
5. Phase (ϕ) : It is defined as the fraction of the time period .

6. Vector or Phase diagram: Taking a reference vector, if we draw various diagrams w.r.t. their angle then it is known as vector or phasor diagram.

Let $i_1 = I_m \, Sin\theta$

$I_2 = I_m \, Sin\,(\theta+\alpha)$

$i_1 = I_m \, Sin\,(\theta-\beta)$



Equation of Alternating emf & current:

Since we understand the diagram of an a.c. is like a sinusoidal wave form. But question arises why does it take the sine form instead of other form? So to know its shape w must have to derive a mathematical formula from where we can confirm the shape of an a.c.The derivation is given bellow.

Let

Φ_m =Maximum flux, in wb

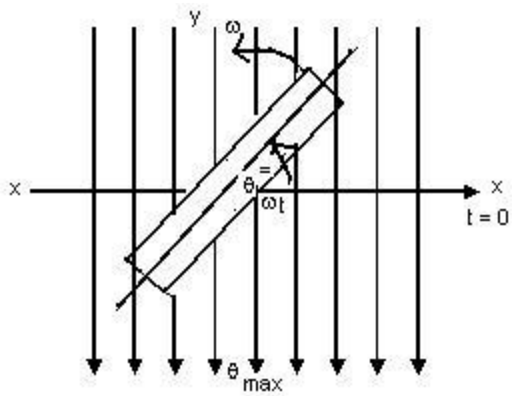
B_m =Maximum flux density, in wb/m^2

ω=Angular velocity of the conductor ,in rad/s

θ=Angular shift of the conductor ,in rad

N= No of conductors or turns in a coil.

A= Area of the conductor, m^2



Generation of Alternating E.M.F.

Let's consider a uniform magnetic field in which fluxes are coming from north pole to south pole .Let's assume a conductor was placed parallel to the x-axis (i.e. θ=0). Now let's that conductor be displaced an angle θ .The maximum flux at this position will have two components, one is along the plane of the conductor & other is normal to the plane of the conductor. But the component which is normal to the plane of the conductor is most effective.

Hence effective component of flux $\phi = \Phi_m \cos\theta$, wb

According to the Faradays laws of Electro Magnetic Induction,

Emf induced in N numbers of conductor = $-N \frac{d\emptyset}{dt}$

i.e. $e = -N \frac{d(\emptyset m \cos\theta)}{dt}$

$= -N \frac{d(\emptyset m \cos\omega t)}{dt}$

$= N\omega \Phi_m \sin\omega t$

$= \omega N \Phi_m \sin\theta$

Hence instantaneous induced emf is given by $\boxed{e = E_m \sin\theta}$

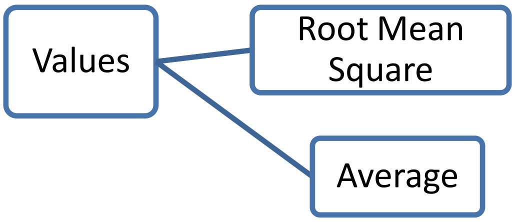
Where $E_m = \omega N \Phi_m$ & is called as maximum induced emf.

Similarly,

Instantaneous current equation is given by $\boxed{i = I_m \sin\theta}$

Instantaneous Voltage equation is given by $\boxed{v = V_m \sin\theta}$

Values of an Alternating Current:



Root Mean Square Value:

Definition : It is defined as the virtual or imaginary value which produces same heating effect while passing through a resistance for certain time period as that of produced by an alternating current passing through the same resistance for time period .

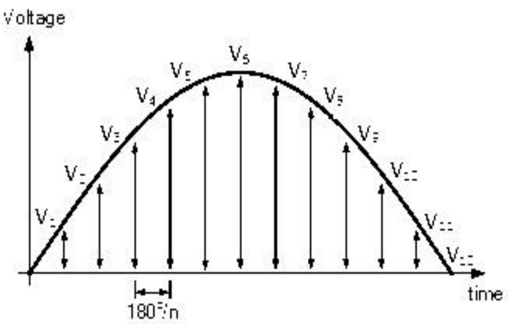
It can be calculated in two different methods such as

a) Mid-Ordinate Method
b) Analytical Method

Mid-Ordinate Method:

Definition: This is a popular method for finding R.M.S value of an A.C.

Explanation:



Let's consider a +ve half cycle of voltage sinusoidal wave which takes t second.
Let it be divided in to n numbers of equal intervals. So each interval takes t/n second.
According to the Joules law of heating, Heat produced in a resistance, H= $i^2 \frac{Rt}{J}$

So heat produced due to flow of i_1 current h1 =$i_1{}^2 \frac{Rt}{Jn}$

Heat produced due to flow of i_2 current, h_2= $i_2{}^2 \frac{Rt}{Jn}$

Heat produced due to flow of i_3 current, h_3= $i_3{}^2 \frac{Rt}{Jn}$

Heat produced due to flow of i_4 current, h_4= $i_4{}^2 \frac{Rt}{Jn}$

Heat produced due to flow of i_n current, h_n= $i_n{}^2 \frac{Rt}{Jn}$

Hence total heat produced by the resistance R during +ve half cycle of a.c. is given by

$H=h_1 + h_2 + h_3 + h_4 +.........+ h_n$

$=\frac{Rt}{Jn} [i_1^2 + i_2^2 + i_3^2 + i_4^2 +........ + i_n^2]$

But heat produced in the same resistance due to flow of a direct current for same time period that of a.c. is given by

$H = I^2Rt/j$

According to the definition of RMS value of an a.c.

$H^{|} = H$

$I^2Rt/j = \frac{Rt}{Jn}$ [$i_1^2 + i_2^2 + i_3^2 + i_4^2 + \dots\dots\dots + i_n^2$]

So $I^2 = \frac{1}{n}$ [$i_1^2 + i_2^2 + i_3^2 + i_4^2 + \dots\dots\dots + i_n^2$]

So $I = \sqrt{\frac{1}{n}}$ [$i_1^2 + i_2^2 + i_3^2 + i_4^2 + \dots\dots\dots + i_n^2$]

Hence

$I_{RMS} = \sqrt{\frac{1}{n}}$ [$i_1^2 + i_2^2 + i_3^2 + i_4^2 + \dots\dots\dots + i_n^2$]

Similarly, V_{RMS} & E_{RMS} can be written accordingly.

Analytical Method for finding RMS value of an a.c:

Equation of current sine wave is

$i = I_m \sin\theta$

Squaring both sides we get (RMS means - S-square, M-mean/avg ,R-root over)

$I^2 = I_m^2 \sin^2\theta$ (mean=sum /period = integration of current(I) function/period)

$= I_m^2 (1+ \cos 2\theta)/2$

Taking mean period, we get

Mean value $= \int_0^{2\Pi} \frac{I^2 d\theta}{2\Pi}$

$= \int_0^{2\Pi} \frac{I_m^2 \sin^2\theta \, d\theta}{2\Pi}$

$= \int_0^{2\Pi} \frac{I_m^2}{4\Pi} (1+\cos 2\theta) \, d\theta$

$= \frac{I_m^2}{4\Pi} \{ \int_0^{2\Pi} d\theta + \int_0^{2\Pi} Cos2\theta \, d\theta \}$

$= \frac{I_m^2}{4\Pi} \{ [\theta]_0^{2\Pi} + [\sin 2\theta]_0^{2\Pi} \}$

$= \frac{I_m^2}{4\Pi} \{ 2\Pi + 0 \}$

$= \frac{I_m^2}{4\Pi} X\ 2\Pi$

$= I_m^2/2$

$\sqrt{\int_0^{2\Pi} \frac{I2}{2\Pi}} = \sqrt{I_m^2/2}$

Hence $I_{RMS} = \frac{Im}{\sqrt{2}}$

Similarly $V = \frac{Vm}{\sqrt{2}}$ & $E = \frac{Em}{\sqrt{2}}$

Average Value:

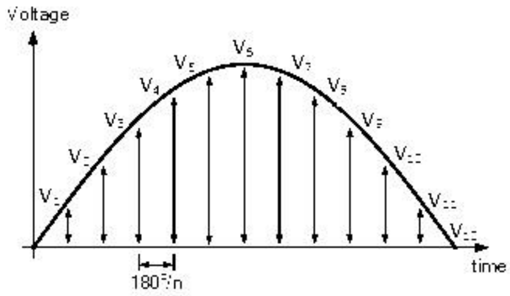
Definition: It is defined as a mean value of an a.c. which is also equal to the ratio between area and base of the wave.

Mathematically, Average value $= \frac{Area\ of\ the\ curve}{Base}$

It can be derived in two different methods such as

a) Mid-Ordinate Method
b) Analytical Method

Mid Ordinate Method:



Let's consider a +ve half cycle of voltage sinusoidal wave which takes t second. Let it be divided in to n numbers of equal intervals. So each interval takes t/n second.

According to the definition, $V_{av} = \frac{Sum\ of\ Areas\ of\ the\ curve}{Base}$

$$= \frac{(V1Xt/n) + (V2Xt/n) + -- + (VnXt/n)}{t}$$

$$= t/n\ [V_1 + V_2 + V_3 + ... + Vn]/t$$

$$V_{av} = \frac{[V1 + V2 + V3 + ... + Vn]}{n}$$

Similarly $I_{av} = \frac{I1 + I2 + I3 + ...+ In}{n}$

Analytical Method for finding Avg value of an A.C.:

Since average value of a sinusoidal wave over one complete cycle is zero, so derivation is done for half cycle only.

Equation of current sine wave, $i = I_m \, Sin\theta$

Taking mean period of the next repeat, we get

Integrating with existing range we get,

$$\int_0^{\Pi} \frac{id\theta}{\Pi} = \int_0^{\Pi} \frac{Im\ Sin\theta\ d\theta}{\pi}$$

$$= \frac{Im}{\Pi} \int_0^{\Pi} Sin\theta\ d\theta$$

$$= \frac{Im}{\Pi} [-Cos\theta]_0^{\Pi}$$

$$= -\frac{Im}{\Pi} [Cos\ \Pi - Cos0]$$

$$= -\frac{Im}{\Pi} [-1 -1]$$

Hence $\boxed{I_{av} = 2\frac{Im}{\Pi}}$

Important Terminology used in Alternating Current:

1. Form Factor (K_f) : It is defined as the ratio between RMS value to the Average value of an ac.

$$K_f = \frac{RMS\ Value}{Average\ Value}$$

For an a.c. $K_f = 1.11$

2. Crest/ Amplitude/Peak Factor:

Definition: It is defined as the ratio of maximum values to the RMS value of an a.c.

Mathematically, Peak Factor, K_P $=\frac{Maximum\ Value}{RMS\ Value}$

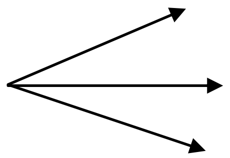
3. Phase (ϕ): The fraction of time period is known as Phase. Basically with respect to phase we use two words like Lagging & Leading.

The phasor or vector diagram of lagging & leading quantities are shown below.

Let $i_1 = I_{m1}\ Sin\theta$

$I_2 = I_{m2}\ Sin(\theta + \alpha)$ Leading i_2

$I_3 = I_{m3}\ Sin(\theta - \beta)$ Lagging I_1



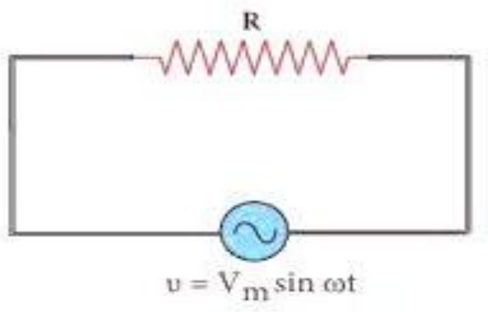
A.C. through Pure Loads:

In this article we will discuss what happens to the alternating current when an alternating voltage is supplied to following three Pure Electric loads.

1. A.C. Through Pure Resistance
2. A.C. Through Pure Inductance
3. A.C. Through Pure Capacitance.

1. A.C. Through Pure Resistance:

Circuit Diagram:



Let,

i= Instantaneous current in the circuit, A

v= Instantaneous Supplied voltage of the circuit, V

I=RMS value of current, A

V=RMS value of supplied voltage, V

I_m= Maximum current in the circuit, A

V_m= Maximum voltage of the circuit, V

Derivation for instantaneous current:

In the above circuit diagram, we can write

$$i=\frac{v}{R}=\frac{Vm\ Sin\theta}{R}=I_m\ Sin\theta$$

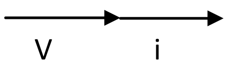
Hence equation of current is $i= I_m\ Sin\theta$

Phasor/Vector diagram:

In case of a pure resistance, If $i= I_m\ Sin\theta$

Then, $v= V_m\ Sin\theta$

Taking instantaneous voltage as reference vector, the current vector is drawn bellow.



Electric Power Derivation:

We have,

$v = V_m \, Sin\theta$

$i = I_m \, Sin\theta$

Instantaneous power, $p = v \; i$

$= V_m \, Sin\theta \; I_m \, Sin\theta$

$= V_m \, I_m \, Sin^2\theta$

$= V_m \, I_m \, [1 - Cos2\theta\,]/2$

$= \frac{VmIm}{2}[1 - Cos2\theta]$

Now average power over one complete cycle is given by

$P = \frac{1}{2\Pi}\int_0^{2\Pi} p \; d\theta$

$= \frac{VmIm}{4\Pi}\{ \int_0^{2\Pi} d\theta \; + \; \int_0^{2\Pi} Cos2\theta \; d\theta \; \}$

$= \frac{VmIm}{4\Pi} \; \{ \; [\theta]_0^{2\Pi} \; + [\; Sin2\theta/2]_0^{2\Pi} \; \}$

$= \frac{VmIm}{4\Pi} \; \{ (2\Pi - 0) \; + 0 \; \}$

$= \frac{VmIm}{4\Pi} \; X \; 2\Pi$

$= \frac{VmIm}{2}$

$= \frac{Vm}{\sqrt{2}} \; x \; \frac{Im}{\sqrt{2}}$

$= V_{RMS} \; x \; I_{RMS}$

Hence total power consumed by a pure Resistance is given by

$\boxed{P = VI}$ W

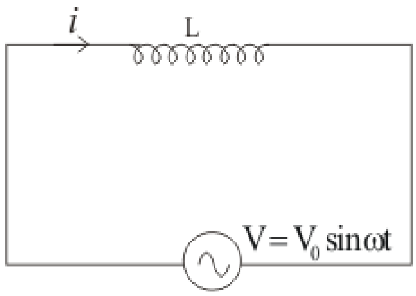
Conclusion:

In case a pure resistance following important conclusions can be remembered.

a) Current is always i-phase or in same phase with the voltage
b) Phase angle between voltage & current is zero i.e. $\phi=0$
c) Hence power factor, $Cos\phi$ is unity
d) Total power delivered by the source or consumed by the load is product of RMS values of voltage & current.

2. A.C. Through Pure Inductance:

Circuit Diagram:



Let,

i= Instantaneous current in the circuit, A

v= Instantaneous Supplied voltage of the circuit, V

I=RMS value of current, A

V=RMS value of supplied voltage, V

I_m= Maximum current in the circuit, A

V_m= Maximum voltage of the circuit, V

L=Inductance of the circuit,H

$X_L = \omega L = 2\Pi f L =$ Reactance offered by the Inductor or Inductive Reactance, Ω

Derivation for instantaneous current:

In the above circuit diagram, we can write

$V_L = v$

$L\frac{di}{dt} = V_m \text{ Sin}\theta$

$\frac{di}{dt} = \frac{Vm}{L} \text{Sin}\theta$

$di = \frac{Vm}{L} \text{Sin}\theta \ dt$

$= \frac{Vm}{L} \text{Sin}\omega t \ dt$

Integrating both sides we get,

$\int di = \frac{Vm}{L} \int \text{Sin}\omega t \ dt$

$i = \frac{Vm}{L} \ (-\text{Cos}\omega t/\omega)$

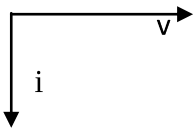
$i = \frac{Vm}{\omega L} (-\text{Sin}\ (90-\theta))$

$= \frac{Vm}{XL} \text{Sin}\ (\theta-90)$

Hence equation of instantaneous current is given by

$\boxed{i = I_m \text{ Sin}\ (\theta-90)}$ A

Phasor/Vector diagram:



In case of a pure Inductance, If $v = V_m \text{ Sin}\theta$

Then $i = I_m \text{ Sin}\ (\theta-90)$

Taking instantaneous voltage as reference vector, the current vector is drawn bellow.

Electric Power Derivation:

We have,

$v = V_m \text{ Sin}\theta$

$i = I_m \text{ Sin}\ (\theta-90) = -I_m \text{ Cos}\theta$

Instantaneous power, p= v X i

$= V_m \sin\theta \ X - I_m \cos\theta$

$= -V_m I_m \sin\theta \cos\theta$

$= -\frac{VmIm}{2}[\sin 2\theta]$

Now average power over one complete cycle is given by

$P=\frac{1}{2\Pi}\int_0^{2\Pi} p\, d\theta$

$=\frac{-VmIm}{4\Pi}\{\int_0^{2\Pi} Sin2\theta d\theta \}$

$=-\frac{VmIm}{4\Pi}\{ -\cos\ 2\theta|_0^{2\Pi}\}/2$

$=\frac{VmIm}{8\Pi}\{ \text{Cos}4\Pi\text{-Cos}0 \}$

$=\frac{VmIm}{8\Pi}$ X 0

= 0

P=0

Hence total power consumed by a pure inductance is given by P=0.

Conclusion:

In case a pure Inductance following important conclusions can be remembered.

1. Current is always Lags behind the voltage by an angle 90^0 or $\frac{\Pi}{2}$ rad
2. Phase angle between voltage & current is 90^0 or $\frac{\Pi}{2}$ rad i.e. $\phi=90^0$ or $\frac{\Pi}{2}$ rad
3. Hence power factor, Cosϕ is zero
4. Total power delivered by the source or consumed by the load is zero

2. A.C. Through Pure Capacitance :

Let,

i= Instantaneous current in the circuit, A

v= Instantaneous Supplied voltage of the circuit, V

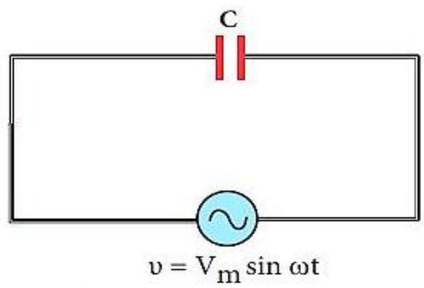
I=RMS value of current, A

V=RMS value of supplied voltage, V

I_m= Maximum current in the circuit, A

V_m= Maximum voltage of the circuit, V

C=Capacitance of the circuit, F



$X_c= \frac{1}{\omega C}= \frac{1}{2\Pi f C}=$ Reactance offered by the capacitor or Capacitive Reactance, Ω

Derivation for instantaneous current:

In the above circuit diagram, we can write

V_c= v

$\frac{q}{C} = V_m \sin\theta$

$q = CV_m \sin\theta$

Differentiating w.r.t time we get

$\frac{dq}{dt} = CV_m \frac{d}{dt}(\sin\theta)$

$i=\omega CV_m \cos\theta$

$= \frac{Vm}{1/\omega C}$ Sin(90+θ)

$= \frac{Vm}{Xc}$ Sin (90+θ)

$= I_m$ Sin (θ+90)

Hence equation of instantaneous current is given by

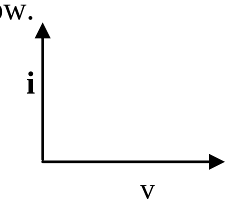
$$\boxed{i = I_m \text{ Sin } (\theta+90)} \text{ A}$$

Phasor/Vector diagram:

In case of a pure Inductance, If $v = V_m \text{ Sin}\theta$

Then $i = I_m \text{ Sin } (\theta+90)$

Taking instantaneous voltage as reference vector, the current vector is drawn bellow.



Electric Power Derivation:

We have,

$v = V_m \text{ Sin}\theta$

$i = I_m \text{ Cos}\theta$

Instantaneous power, p= v i

$= V_m \text{ Sin}\theta \; I_m \text{ Cos}\theta$

$= -V_m I_m \text{ Sin}\theta \text{ Cos}\theta$

$= \frac{VmIm}{2}[\text{Sin}2\theta]$

Now average power over one complete cycle is given by

$P = \frac{1}{2\Pi}\int_0^{2\Pi} p \, d\theta$

$= \frac{VmIm}{4\Pi}\{\int_0^{2\Pi} Sin2\theta d\theta \}$

$= -\frac{VmIm}{4\Pi}\{\cos 2\theta|_0^{2\Pi}\}/2$

$= \frac{-VmIm}{8\Pi}$ { Cos4Π-Cos0 }

$= -\frac{VmIm}{8\Pi}$ X 0

= 0

Hence total power consumed by a pure capacitance is given by

$$\boxed{P = 0} \text{ W}$$

Conclusion:

In case a pure Capacitance following important conclusions can be remembered.

1. Current is always Leads the voltage by an angle 90^0 or $\frac{\Pi}{2}$ rad
2. Phase angle between voltage & current is 90^0 or $\frac{\Pi}{2}$ rad i.e. $\phi = 90^0$ or $\frac{\Pi}{2}$ rad
3. Hence power factor, Cosϕ is zero
4. Total power delivered by the source or consumed by the load is zero

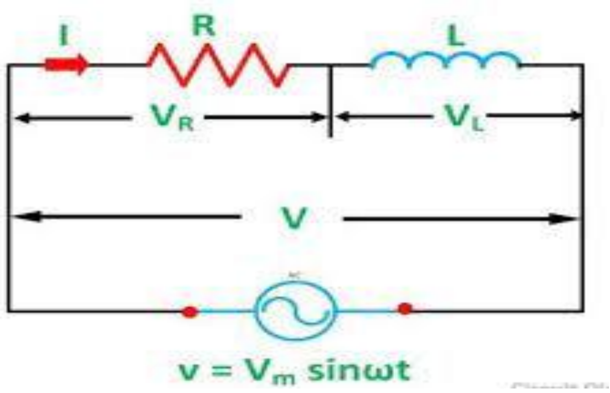
A.C. series circuit:

In this article we will discuss following three series circuit & few important relations.

1. R-L Series Circuit
2. R-C Series Circuit
3. R-L-C Series Circuit

5.1 A.C. through R-L, R-C & R-L-C Circuit:

R-L Series Circuit:



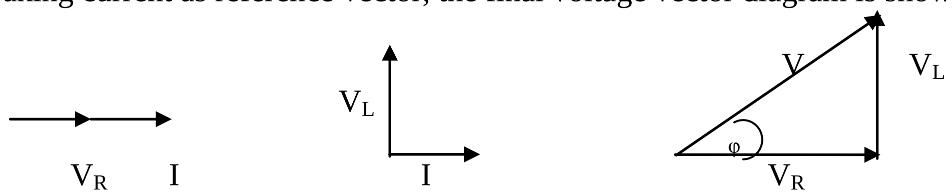
Let,
I= Total or Net or Circuit or Supply current, A
V= Total or Net or Circuit or Supply Voltage, V
I_m= Maximum Current, A
V_m=Maximum Voltage, V
V_R = Voltage across the resistance, V
V_L= Voltage across the Inductor, V
I_R=Current through the Resistance, A
I_L=Current through the Inductor, A

Voltage Vector Diagram:

From the above circuit diagram we can write,

$\vec{V} = \overrightarrow{VR} + \overrightarrow{VL}$

But Current in the series circuit remains the same i.e. I= I_R =I_L
Taking current as reference vector, the final voltage vector diagram is shown below. Fig



(For Resistance only) (For Inductance only) (Resultant voltage diagram)

Abstract of Voltage Triangle:

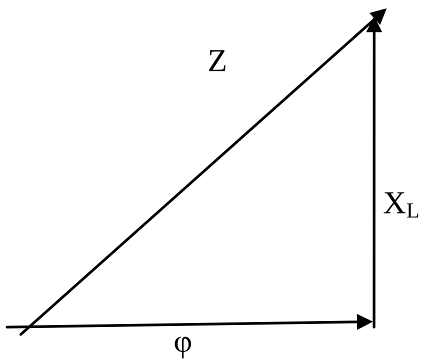
Angle between V& I is ϕ & is called is power factor (p.f.) angle or phase angle.

$\vec{V} = \overrightarrow{VR} + J\ \overrightarrow{VL}$

$|\vec{V}| = \sqrt{(V_R^2 + V_L^2)}$

Impedance Triangle:

Abstract of Impedance Triangle:



$Z= R+ JX_L$

$|z| = \sqrt{(R^2 + X_L^2)}$

$R = Z\ Cos\phi$ & $X_L = Z\ Sin\phi$

Power Derivation:

In the voltage vector diagram, it is observed that,

$v = V_m \sin\theta$

$i = I_m \sin(\theta - \phi)$

Instantaneous power, p= v X i

$$= V_m \sin\theta \times I_m \sin(\theta-\phi)$$

$$= \frac{V_m I_m}{2} \; 2 \sin\theta \sin(\theta-\phi)$$

$$= \frac{V_m I_m}{2} [\cos(\phi) - \cos(2\theta-\phi)]$$

Total or Average Power over one complete cycle is given by

$$P = \frac{1}{2\Pi}\int_0^{2\Pi} p \, d\theta$$

$$= \frac{V_m I_m}{4\Pi} \left[\int_0^{2\Pi} \cos\phi \, d\theta + \int_0^{2\Pi} \cos(2\theta - \phi) \, d\theta\right]$$

$$= \frac{V_m I_m}{4\Pi} [\cos\phi \times 2\Pi + 0]$$

$$= \frac{V_m I_m}{2} \cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{V_m I_m}{\sqrt{2}} \cos\phi$$

$$= V_{RMS} \times I_{RMS} \cos\phi$$

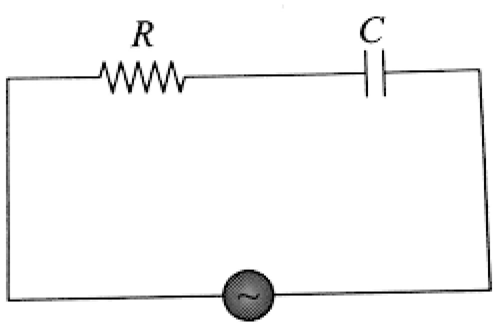
Hence Power consumed is P= V I Cosϕ W

Conclusion:

In case a R-L series Circuit, following important conclusions can be remembered.

a) Current is always Lags the voltage by an angle ϕ ($0 < \phi < 90$)
b) Total power delivered by the source or consumed by the load V I Cosϕ
c) Cosϕ is called as power factor of the circuit.

R-C Series Circuit:



Let,

I= Total or Net or Circuit or Supply current, A

V= Total or Net or Circuit or Supply Voltage, V

I_m= Maximum Current, A

V_m=Maximum Voltage, V

V_R = Voltage across the resistance, V

V_C= Voltage across the Capacitor, V

I_R=Current through the Resistance, A

I_C=Current through the Capacitor, A

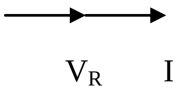
Voltage Vector Diagram:

From the above circuit diagram we can write,

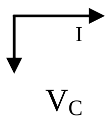
$\vec{V} = \vec{V}r + \vec{Vc}$

But Current in the series circuit remains the same i.e. $I = I_R = I_L$

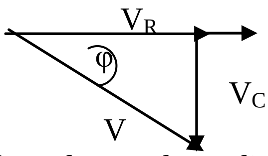
Taking current as reference vector , the final voltage vector diagram is shown below.



(For Resistance only)



(For Inductance only)



(Resultant voltage diagram)

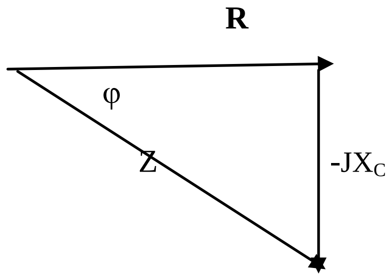
Angle between V& I is ϕ & is called is power factor (p.f.) angle or phase angle.

$\vec{V} = \overrightarrow{VR} - J\ \overrightarrow{Vc}$

$|\vec{V}| = \sqrt{(V_R^2 + Vc^2)}$

Impedance Triangle:

Abstract of Impedance Triangle:



$Z = R - jx_C$

$|z| = \sqrt{(R^2 + X_C^2)}$

$R = Z\ Cos\phi\ \ \&\ \ X_C = Z\ Sin\phi$

Power Derivation:

In the voltage vector diagram, it is observed that,

$v = V_m\ Sin\theta$

$i = I_m\ Sin(\theta + \phi)$

Instantaneous power, $p = v \times i$

$$= V_m\ Sin\theta \times I_m\ Sin(\theta+\phi)$$

$$= \frac{V_m I_m}{2}\ 2\ Sin\theta\ Sin(\theta+\phi)$$

$$= \frac{V_m I_m}{2}\ [\ Cos\ (-\phi) - Cos\ (2\theta-\phi)\ ]$$

Total or Average Power over one complete cycle is given by

$$P = \frac{1}{2\Pi}\int_0^{2\Pi} p\ d\theta$$

$$= \frac{V_m I_m}{4\Pi}\ [\ \int_0^{2\Pi} Cos\phi\ d\theta\ + \int_0^{2\Pi} Cos\ (2\theta - \phi)\ d\theta\ ]$$

$$= \frac{V_m I_m}{4\Pi}\ [\ Cos\phi \times 2\Pi\ +0\ ]$$

$$= \frac{V_m I_m}{2}\ Cos\phi$$

$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$ Cosϕ

$= V_{RMS} \times I_{RMS}$ Cosϕ

Hence Power consumed is P= V I Cosϕ

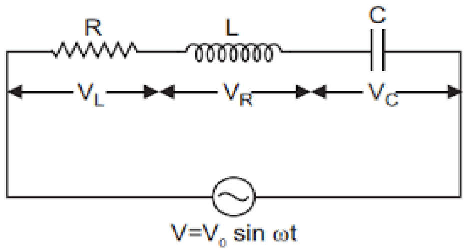
Conclusion:

In case a R-C series Circuit, following important conclusions can be remembered.

d) Current is always Leads the voltage by an angle ϕ ($\phi > 270$ & < 360)
e) Total power delivered by the source or consumed by the load V I Cosϕ
f) Cosϕ is called as power factor of the circuit.

R-L-C Series Circuit:

Circuit Diagram:

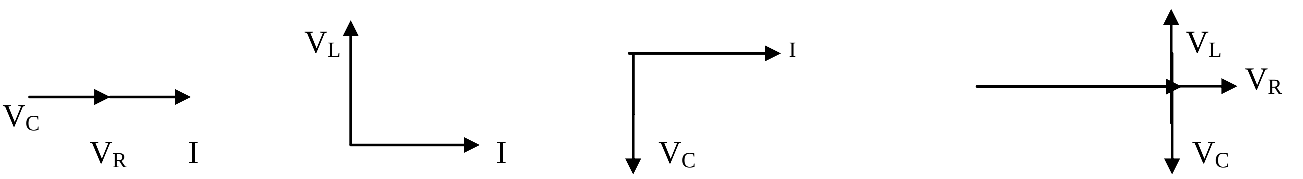


(a)

Let,

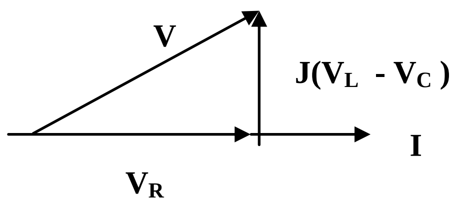
I= Total or Net or Circuit or Supply current, A
V= Total or Net or Circuit or Supply Voltage, V
I_m= Maximum Current, A
V_m=Maximum Voltage, V
V_R = Voltage across the resistance, V
V_L= Voltage across the Inductor, V
V_C= Voltage across the Capacitor, V
I_R=Current through the Resistance, A
I_L=Current through the Inductor, A
I_C=Current through the Capacitor, A

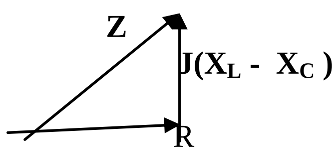
But Current in the series circuit remains the same i.e. I= I_R =I_L =I_C
Taking current as reference vector, the final voltage vector diagram is shown below.



(For Resistance only) (For Inductance only) (For Capacitance only) (Resultant voltage diagram)

Case-I (If $V_L > V_C$)





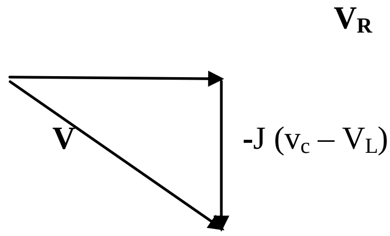
For impedance triangle ($X_L > X_C$)

i)Net reactance X= (X_L - X_C)

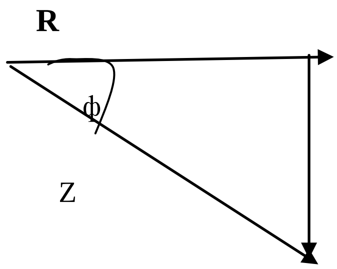
ii) Impedance Z = R + J (X_L - X_C)

iii) [Z] = { √ R^2 + (X_l –X_c)^2 }

Case-II (If V_C > V_L)
Voltage vector diagram is shown in the fig.

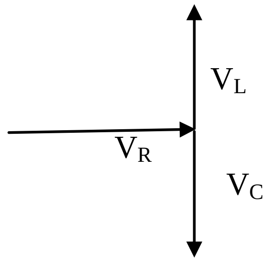


For Impedance Triangle (X_C > X_L)
i)Net reactance X= (X_C - X_L)
ii) Impedance Z = R + J (X_C - X_L)
iii) [Z] = √{R^2 + (X_C –X_L)^2 }-J(X_C –X_L)



Case-II (If V_C =V_L)

Voltage vector diagram is shown in the fig.



Net voltage = V_R

Similarly, since X_L = X_C so net reactance is zero.

Hence Z = R

5.2 Solution of problems of A.C. through R-L, R-C & R-L-C series Circuit by complex algebra method:

Phasor Algebra: →

A vector quantity can be expressed in terms of

(i) Rectangular or Cartesian form

(ii) Trigonometric form

(iii) Exponential form

(iv) Polar form

If we write AC quantity in rectangular form like E = a + jb

$$E = a + jb$$
$$= E(\cos\theta + j\sin\theta)$$

Where a $= E\cos\theta$ is the active part

b $= E\sin\theta$ is the reactive part

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$j = \sqrt{-1}(90°)$

$j^2 = -1(180°)$

$j^3 = -j(270°)$

$j^4 = 1(360°)$

angle $j = \sqrt{-1}(90°)$

(i) Rectangular for :-

$$E = a \pm jb$$

$$\tan\theta = b/a$$

(ii) Trigonometric form :-

$$E = E(\cos\theta \pm j\sin\theta)$$

(iii) Exponential form :-

$$E = Ee^{\pm j\theta}$$

(iv) Polar form :-

$$E = E/\pm e \ \left(E = \sqrt{a^2 + b^2}\right)$$

Addition or Subtraction:-

$$E_1 \quad = a_1 + jb_1$$

$$E_2 \quad = a_2 + jb_2$$

$$E_1 \pm E_2 \quad = (a_1 + a_2) \pm (b_1 + b_2$$

$$\phi \quad = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$$

Multiplication :-

$$E_1 \times E_2 \quad = (a_1 + ja_1) \pm (a_1 + jb_2)$$

$$= (a_1a_2 - b_1b_2) + j(a_1a_2 + b_1b_2)$$

$$\phi \quad = \tan^{-1}\left(\frac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2}\right)$$

$$E_1 \quad = E_1\angle\theta_1$$

$$E_2 \quad = E_2\angle\theta_2$$

$$\mathrm{E}_1 \times \mathrm{E}_2 = \mathrm{E}_1\mathrm{E}_2 \angle\phi_1 + \phi_2$$

Division $\quad : -$

$$E_1 \quad = E_1\angle\theta_1$$

$$E_2 \quad = E_2\angle\theta_2$$

$$\frac{E_1}{E_2} \quad = \frac{E_1\angle\theta_1}{E_2\angle\theta_2} = \frac{E_1}{E_2}\angle\theta_1 - \theta_2$$

<u>Problem 1:</u>

Covert the rectangular form of $(3 + j4)$ into polar form

<u>solution</u>.

Let $\mathrm{P} = 3 + \mathrm{j}4$

Magnitude $|P| = \sqrt{3^2 + 4^2} = 5$

Angle ' θ ' $= \tan^{-1}\left(\frac{4}{3}\right)$

$= \tan^{-1}(1.33)$

$= 53 \cdot 13°$

$\therefore$ ' p ' in polar form $= 5\angle 53 \cdot 13°$.

<u>Problem 2:</u>

Convert the polar form of (i) $10\angle 30°$ (ii) $50\angle -60°$ into rectangular form.

Solution:

(i) Here magnitude is 10 and phase angle is $30°$.

$\therefore$ Real component $= 10\cos 30°$

$= 10 \times 0.866 = 8.66$

Quadrature component $= 10\sin 30 = 10 \times \frac{1}{2} = 5$

$\therefore$ Rectangular form $= (8.66 + \text{j}5)$.

(ii) $50\angle -60°$

Here the magnitude is 50 and phase angle is $-60°$.

$\therefore$ Real component $= 50\cos(-60) = 50 \times \frac{1}{2} = 25$

Quadrature component $= 50\sin(-60) = 50 \times (-0.866)$

$= -43.30$

$\therefore$ Rectangular form $= 25 - \text{j}43.3$ (Ans)

Problem 3:

A series R, L, C circuit with $R = 10\Omega, L = 100\text{mH}$ and $C = 200\mu F$ connected across an ac source of frequency 50 Hz. Find the impedance in complex form and its magnitude. Draw the impedance triangle and find the phase angle between voltage and current phasor?

Solution

Given $\text{R} = 10\Omega, \text{L} = 100\text{mH}$

$= 100 \times 10^{-3}\text{H}$

$\text{C} = 200\mu\text{F} = 200 \times 10^{-6}$ F.

f $= 50$ Hz.

$\therefore \text{Z} = \text{R} + \text{jX}$ (in complex form)

$\text{X}_\text{L} = \omega\text{L} = 2\pi\text{fL}$

$= 2 \times \pi \times 50 \times 100 \times 10^{-3}$

$= 10\pi = 31.4\Omega$

$$\therefore \text{X}_\text{C} = \frac{1}{\omega\text{C}} = \frac{1}{2\pi\text{fc}} = \frac{1}{2 \times 3 \cdot 14 \times 50 \times 200 \times 16^{-6}}$$

5.3 Solution of problems of A.C. through R-L, R-C & R-L-C parallel & Composite Circuits

Problem 1:

How is a current of 10A shared among three circuits in parallel, the impedances are $(2 - j5)\Omega$, $(6 + j3)\Omega$ and $(3 + j4)\Omega$ respectively.

Solution:.

Given $Z_1 = (2 - j5)\Omega, Z_2 = (6 + j3)\Omega,, Z_3 = (3 + j4)\Omega$

In polar form $Z_1 = 5 \cdot 39\angle -68 \cdot 2°$

$Z_2 = 6 \cdot 71\angle 26 \cdot 6°$

$Z_3 = 5\angle 53°$

Total admittance $y = y_1 + y_2 + y_3$

But $y_1 = \frac{1}{Z_1} = \frac{1}{5\cdot 39\angle -68\cdot 2} = 0\cdot 186\angle 68\cdot 2°$

$$y_2 = \frac{1}{Z_2} = \frac{1}{6\cdot 71\angle 26\cdot 6°} = 0\cdot 149\angle -26\cdot 6°$$

$$y_3 = \frac{1}{Z_3} = \frac{1}{5\angle 53°} = 0\cdot 2\angle -53°$$

$$\therefore y = y_1 + y_2 + y_3 = 0\cdot 322 - j0\cdot 055 = 0.327\angle -9.7°$$

$$I_1 = 10x\frac{y_1}{y} = \frac{10}{0.34} = \frac{10}{0.34} \times 0\cdot 186 = 5 \times 68A$$

$$I_2 = 30\cdot 58 \times 0.149 = 4.57A$$

$$I_3 = 30\cdot 58 \times 0.2 = 6\cdot 12A.$$

Problem .A 10 Ω resistance is in series with a 30mH inductance .The series combination is connected to a 200V 50 Hz supply mains, Find

i) Impedance of the circuit
ii) Current in the circuit
iii) Total Power supplied
iv) Total reactive power

Given Data:

R=10 Ω
L=30mH
V=200v
f=50Hz

Required Data:

i) Z= ?
ii) I =?
iii) P=?
iv) P_R= ?

Solution:

i) Inductive Reactance $X_L = 2\Pi f L$

$= 2x\ \Pi\ x\ 50x\ 30x10^{-3} = 9.42\ \Omega$

Impedance of the circuit Z= $\sqrt{(R^2 + X_L^2)}$

$= \sqrt{(10^2 + 9.42^2)}$

= 13.73 Ω Ans

ii) Current in the circuit , I= V/Z

=200/13.73=14.56 A Ans

iii) Power Factor Cosɸ= R/Z

=10/13.73=0.728 (Lag)

Total Power , P= VI Cosɸ

=200X14.56X0.728=2120 W=2.12 KW Ans

iv) Sinɸ=0.686

Hene Reactive Power , P_R= VI Sinɸ=200 X 14.56 X 0.686= 1997.63 VAR=1.997 KV

5.4 Power factor & power triangle:

Power factor (cosφ):

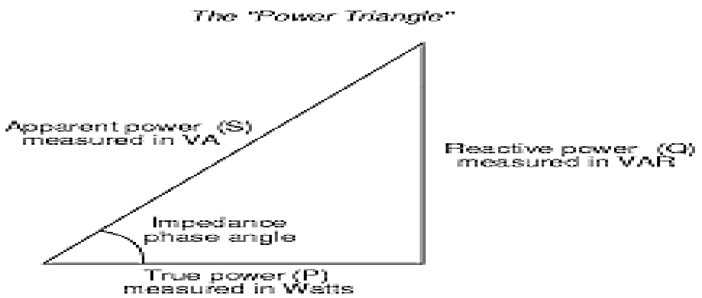
It is defined as the cosine angle between voltage & current is called power factor.
Or it is also defined as the ratio of active (P) to the reactive power (S)

Mathematically

$$\text{Cos } \varphi = \frac{P}{V \times I}$$

$$\text{Power factor } (\cos\varphi) = \frac{Active\ power(P)}{Apparent\ power(S)}$$

Power triangle:



5.5 Deduce expression for active, reactive, apparent power:

Apparent Power (P_{ap}) :

It is the product of RMS value of voltage & current. Its unit is VA. or KVA

$$P= V\ I$$

Active/Real/True/Real Power (P):

The resolved part of apparent power along the real axis is known as the real power. Its unit is W or KW

$$P=VI\ \cos\varphi$$

Reactive Power (P_R) :

The resolved part of apparent power along the Imaginary axis axis is known as the real power. Its unit is VAR or KVAR

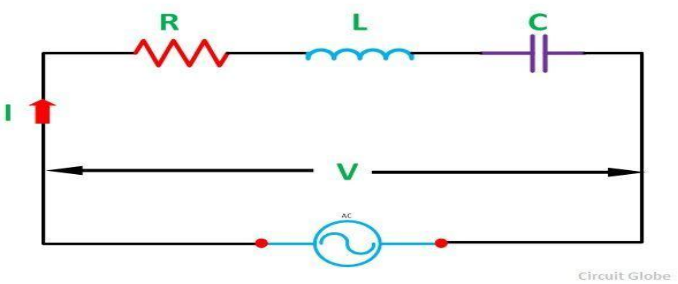
$$P=VI\ \sin\varphi$$

5.6 Derive the resonant frequency of series resonance and parallel resonance circuit:

A.C.SERIES RESONANCE CIRCUIT:

It is defined as the property of an a.c. series circuit in which net reactance is zero.

Or maximum current flows through this circuit



Derivation of Series Resonant Frequency (f_0) :

According to the definition, $X_L = X_C$

$$2\Pi f_0 L = \frac{1}{2\Pi f C}$$

$$f_0^{\,2} = \frac{1}{4\Pi \, X\Pi \, LC}$$

$$f_0 = \frac{1}{2\Pi\sqrt{LC}}$$

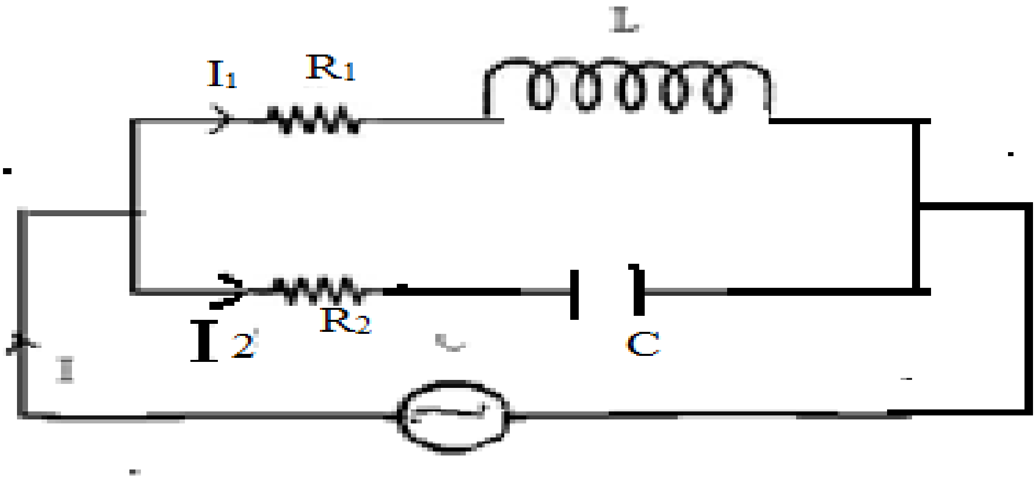
Hence resonant frequency is given by $\boxed{f_0 = \frac{1}{2\Pi\sqrt{LC}}}$

RL & RC parallel ckt:

The parallel resonance condition

When the reactive part of the line current is zero. The net reactance is zero.

The line current will be minimum. The power factor will be unity



When the reactive part of the line current is zero. The net reactance is zero.
The line current will be minimum.
The power factor will be unity

Impedance

$$Z_1 = R_1 + jX_L$$
$$Z_2 = R_2 - jX_C$$

Admittance

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L}$$
$$= \frac{(R_1 + jX_L)}{(R_1 + jX_L)(R_1 - jX_L)}$$
$$= \frac{R_1 + jX_L}{R_1^2 + X_L^2}$$
$$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$$

Admittance

Admittance

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_1 + jX_C}$$
$$= \frac{(R_2 + jX_C)}{(R2_1 - jX_C)(R_2 + jX_C)}$$
$$= \frac{R_2 + jX_L}{R_2{}^2 + X_C{}^2}$$
$$Y_2 = \frac{R_2}{R_2{}^2 + X_C{}^2} + j\frac{X_C}{R_2{}^2 + X_C{}^2}$$

Total Admittance Admittance $\left(\frac{1}{Z}\right) = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$\Rightarrow Y = Y_1 + Y_2$$
$$\Rightarrow Y = \frac{R_1}{R_1{}^2 + X_L{}^2} - j\frac{X_L}{R_1{}^2 + X_L{}^2} + \frac{R2}{R_2{}^2 + X_C{}^2} + j\frac{X_C}{R_2{}^2 + X_C{}^2}$$
$$\Rightarrow Y = \frac{R_1}{R_1{}^2 + X_L{}^2} + \frac{R2}{R_2{}^2 + X_c{}^2} - j\left(\frac{X_L}{R_1{}^2 + X_L{}^2} - \frac{X_C}{R_2{}^2 + X_c{}^2}\right)$$

At Resonance,

$$\frac{X_L}{R_1{}^2 + X_L{}^2} - \frac{X_C}{R_2{}^2 + X_C{}^2} = 0$$
$$\Rightarrow \frac{X_L}{R_1{}^2 + X_L{}^2} = \frac{X_C}{R_2{}^2 + X_C{}^2}$$
$$\Rightarrow X_L(R_2{}^2 + X_C{}^2) = X_C(R_1{}^2 + X_L{}^2)$$
$$\Rightarrow 2\pi fL\left(R_2{}^2 + \frac{1}{4\pi^2 f^2 C^2}\right) = \frac{1}{2\pi fC}(R_1{}^2 + 4\pi^2 f^2 L^2)$$
$$\Rightarrow 2\pi fLR_2{}^2 + \frac{L}{2\pi fC^2} = \frac{R_1{}^2}{2\pi fC} + \frac{2\pi fL^2}{C}$$

$$\Rightarrow \frac{L}{2\pi fc^2} - \frac{R_1{}^2}{2\pi fC} = \frac{2\pi fL^2}{C} - 2\pi fLR_2{}^2$$
$$\Rightarrow \frac{1}{2\pi fC}\left(\frac{L}{C} - R_1{}^2\right) = 2\pi fL\left(\frac{L}{C} - R_2{}^2\right)$$
$$\Rightarrow 4\pi^2 f^2 LC = \frac{\frac{L}{C} - R_1{}^2}{\frac{L}{C} - R_2{}^2}$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}\left(\frac{L - CR_1{}^2}{L - CR_2{}^2}\right)$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{L - CR_1{}^2}{L - CR_2{}^2}\right)}$$

If R_1 and $R_2 = 0$, then

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{L^2 C}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}$$

5.7 Define Bandwidth, Selectivity & Q-factor in series circuit:

Half Power Frequency:

It is defined as a frequency of series a.c. resonance where power delivered to the circuit under resonance will be half of the normal power & frequency.

Hence at half power frequency, respective current is given by

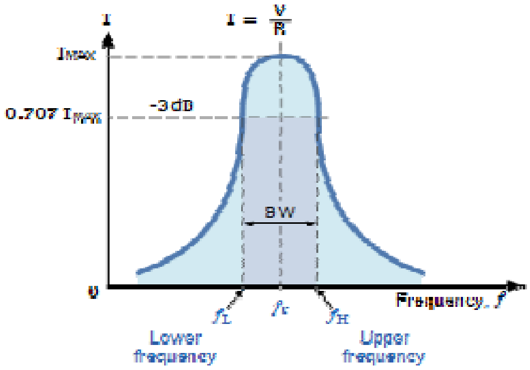
$I_{hp} = I_0/\sqrt{2}$

Band Width:

During half power frequency, two frequencies are observed called as Upper half frequency (f_2) & Lower half frequency (f_1). Hence the difference between these two frequencies is known as band width.

Or $\Delta\omega = \omega_2 - \omega_1$

$\Delta f = f_2 - f_1$



Quality Factor (Q_0) :

It is defined as the ratio between reactive power to the active power.

Or

The ratio of capacitor voltage or inductor voltage at resonant frequency to supply voltage is a measure of quality of a resonance circuit. This term is known as quality factor (Q factor). At the frequency of the resonance (f_0)

$$Q_0 = \frac{2\Pi f0L}{R} = \frac{1}{2\Pi f0RC}$$

Relationship between Q and Bandwidth of R-L-C series circuit:

Bandwidth $= w_2 - w_1$

At $w = w_1$, the reactance is capacity as $X_C \succ X_L$

Hence $\frac{1}{w_1 C} - w_1 L = R$ … … … … .. eq. 1

At $w = w_2$ the reactance is inductive as $X_L \succ X_C$

Hence $w_2 L - \frac{1}{w_2 C} = R$ … … … … … …. eq. 2

From equation 1 we get $w_1^2 LC + w_1 RC - 1 = 0$

Dividing by LC we get $w_1^2 + w_1 \frac{R}{L} - \frac{1}{LC} = 0$

$$w_1 = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

Similarly, from equation 2

$$w_2^2 - \frac{R}{L}w_2 - \frac{1}{LC} = 0$$

$$w_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

Hence bandwidth

$$w_2 - w_1 = \frac{R}{2L} + \frac{R}{2L} = \frac{R}{L}$$

$$\Rightarrow 2\pi(f_2 - f_1) = \frac{R}{L}$$

$$\Rightarrow f_2 - f_1 = \frac{R}{2\pi L} = \frac{f_0}{Q}$$

$$\Rightarrow Q = \frac{f_0}{f_2 - f_1} = \frac{w_0}{BW}$$

5.8 Solve numerical problems:

A 20Ω resistor is connected in series with an inductor, a capacitor and an ammeter across a 25 –v, variable frequency supply. When the frequency is 400Hz, the current is at its Max^{m} value of 0.5 A and the potential difference across the capacitor is 150v. Calculate

(a) The capacitance of the capacitor.

(b) The resistance and inductance of the inductor.

Solution:

Since current is maximum, the circuit is in resonance.

$X_c = V_C / I = 150 / 0.5 = 300\Omega$

(a) $X_C = 1/2\pi f_0 \Rightarrow 300 = 1/2\pi \times 400 \times c$

$\Rightarrow c = 1.325 \times 10^{-6}$ f $= 1.325\mu$f .

(b) $X_L = X_L = 150 / 0.5 = 300\Omega$

(c) $2\pi \times 400 \times L = 300$

$\Rightarrow L = 0.49$H

At resonance,

Circuit resistance = 20+R

$\Rightarrow V/Z = 2510.5$

$\Rightarrow R = 30\Omega$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

Q1.What is Time Period ?

Ans: It is defined as the time taken by an a.c. to form one cycle .It is denoted by T & its unit is second.

Q2.What is frequency of an a.c. ?

Ans: It is defined as the number of cycles formed by an a.c. in one second. It is denoted by f & its unit is Hz.

Mathematically $f=\frac{1}{T}$ Hz or Sec^{-1}

Q3.What is form factor ? [W-05, 17, W-18]

Ans: It is defined as the ratio between RMS values to the Average values of an ac.

$$K_f = \frac{RMS\ Value}{Average\ Value}$$

For an a.c. $K_f = 1.11$

Q4.What is Crest Factor? [W-04]

Ans: It is defined as the ratio of maximum values to the RMS value of an a.c.

Mathematically, Peak Factor, $K_P = \frac{Maximum\ Value}{RMS\ Value}$

Q5.What is power consumed by a pure capacitor ?

Ans . Zero

Q6.What is Power Factor? [W-19]

Ans: It is the ratio between Resistance & Impedance of the circuit. It has no unit.

$Cos\phi = \frac{R}{Z}$

Q7. Define Selectivity? [W-18]

Ans: It is defined as the ratio of resonant frequency to the Half Power Bandwidth.

Q8. Define the Apparent Power? [W-18]

Ans: It is defined as the Product of RMS value of Voltage & Current.

$\boxed{P_{ap} = VI}$ VA

Q9. What do you mean by the resonant frequency?

Ans : The frequency at which net reactance of an a.c. series R-L-C circuit is zero is called as resonant frequency .

Q10.What is Quality Factor of a series resonance circuit? [W-09, 16, 19, 20]

Ans : It is defined as the ratio of Reactance to the ohmic resistance of a series R-L-C circuit.

$$Q_0 = \frac{X}{R}$$

Or, It is the ratio between Resonant frequency & Bandwidth

Q11. What is Resonance? What is condition for resonance of an a.c. R-L-C circuit? [W-12, 14, 15, 17]

Ans: Resonance is the property of an a.c. series circuit in which net reactance is zero.
Conditions:
i) $X_L = X_C$
ii) Net impedance is equal to the Resistance
iii) Power factor is unity.
iv) Current becomes maximum.

Q12.What is the value of form factor of a pure sinusoidal a.c. signal? [W-15, 17, 18]

Ans: 1.11

Q13.What is susceptance?

Ans: Reciprocal of Reactance is called as susceptance.

Q14. What is admittance?

Ans: Reciprocal of Impedance is called as Admittance.

Q15.What is half power frequency? [S-2019]

Ans: The frequency at which power delivered to the series circuit is half of the peak value.

POSSIBLE TYPE LONG QUESTIONS

Q1.Derive power relation of an a.c. series R-L circuit?

Q2. Derive Resonant frequency of an R-L-C series circuit. ?

Q3.A 10 Ω resistance is in series with a 30mH inductance .The series combination is connected to a 200V 50 Hz supply mains, Find

I. Impedance of the circuit
II. Current in the circuit
III. Total Power supplied
IV. Total reactive power

CHAPTER NO.- 09
FILTERS

Learning Objectives:

9.1 Define filter
9.2 Classification of pass Band, stop Band and cut-off frequency.
9.3 Classification of filters.
9.4 Constant – K low pass filter.
9.5 Constant – K high pass filter.
9.6 Constant – K Band pass filter.
9.7 Constant – K Band elimination filter.
9.8 Solve Numerical problems.

9.1 Define filter :

Filter: Filter is an electrical network that can transmit signal within a specified frequency range, this frequency range is called pass band.

Ideal filter: It would transmit signals under the pass band frequencies with out attenuation & completely suppress .

Practical filter: It do not ideally transmit the pass band signal un attenuated, due to absorption ,reflection or due to other losses.

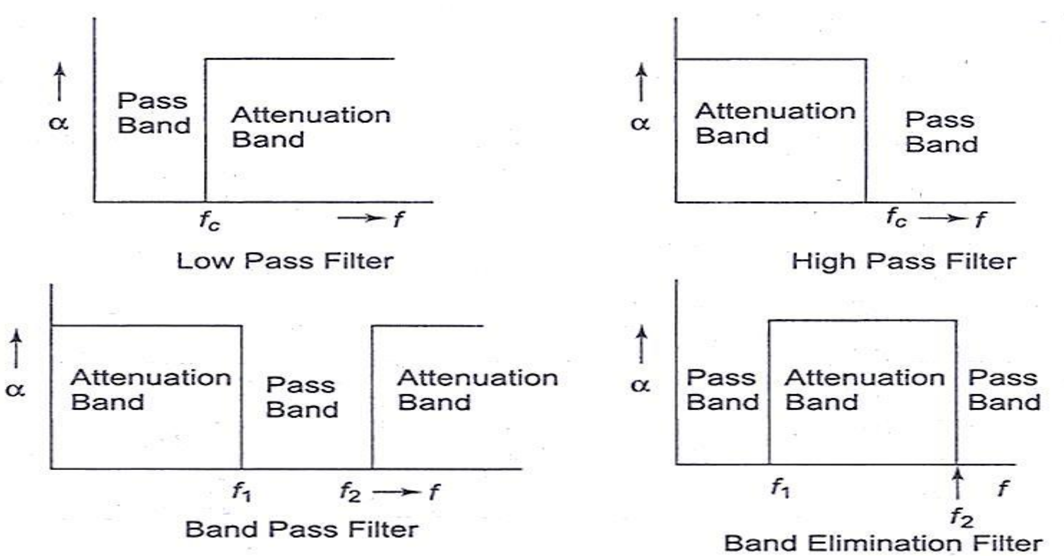
Filter networks are widely used in communication systems to separate various voice channels in carrier frequency telephone circuits, They are classified into four common types, viz .low pass, high pass, band pass and band elimination or band stop .

9.2 Classification of pass Band, stop Band and cut-off frequency:

Pass Band: The frequency band which is transmitted is called as no attenuation band or pass band.

Stop band /Attenuation band: The frequency band which is suppressed is called as attenuation band or stop band

Cut-Off Frequency: The frequency that separate the pass band & attenuation band is called cut-off frequency. It denoted as (f_c)



9.3 Classification of filters:

The filters are classified according to the following two categories.

1. Constant K Filter
2. M Derived Filter.

Constant K filters are divided into the following four types

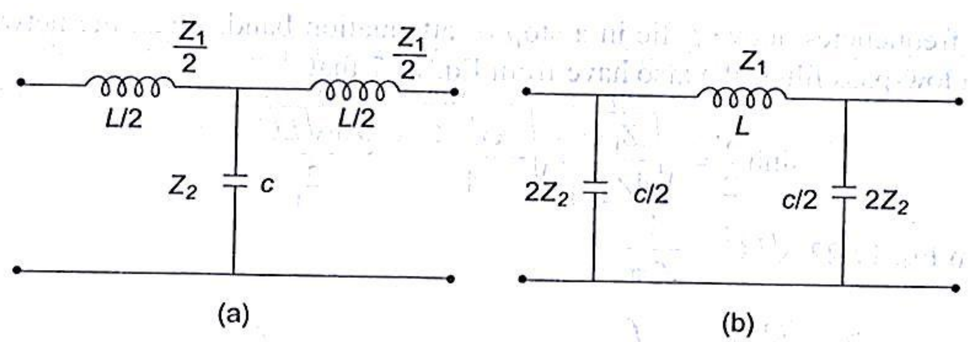
1. Low Pass Filter (LPF)
2. High Pass Filter (HPF)
3. Band Pass Filter (BPF)
4. Band Stop Filter (BSF)

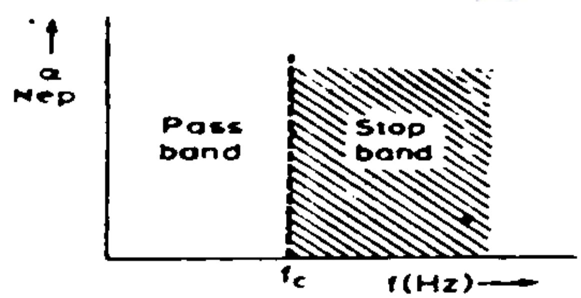
Constant K Filter (Prototype Filter) : This filter is divided into four categories

1. Constant K Low Pass Filter
2. Constant K High Pass Filter
3. Constant K Band Pass Filter
4. Constant K Band Stop Filter

9.4 Constant – K low pass filter:

- It is the simplex type of filter which allows all frequencies up to specified cut-off frequency & attenuates all other frequencies above the cut-off frequency.
- The circuit diagram of low pass filter in 'T' section & 'π' section are shown below

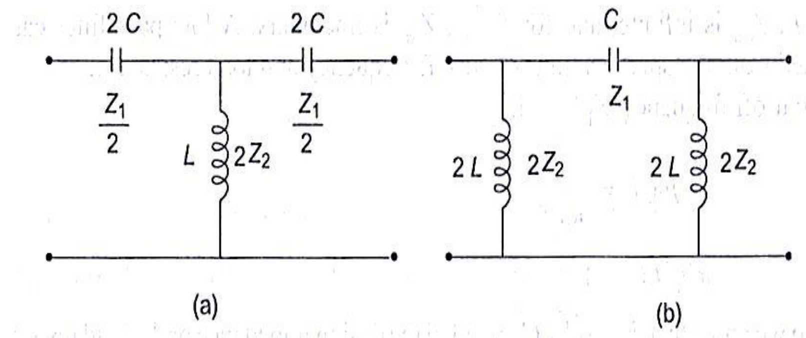


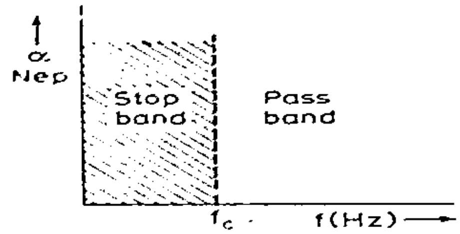


- Cut off frequency, $f_C = \frac{1}{\pi\sqrt{LC}}$
- Cut-off frequency lies between 0 to- $1/(\pi\sqrt{L/C})$
- pass band lies between 0 to- $1/(\pi\sqrt{L/C})$

9.5 Constant – K high pass filter:

- The filter which attenuates all the frequencies below the cut-off frequency & allow to pass all other frequencies above the cut-off frequency.
- The 'T' configuration 'π' configuration of high pass filter are shown on the fig given below.





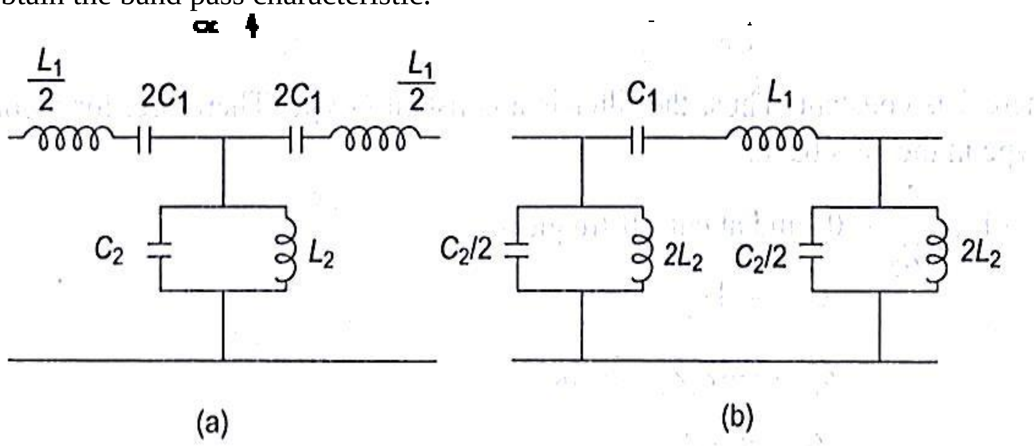
HPF Characteristic

Cut off frequency $f_C = \frac{1}{4\pi\sqrt{LC}}$

- Cut-off frequency lies between 1/(4π$\sqrt{L/C}$) to infinite
- Pass band lies between 1/(4π$\sqrt{L/C}$) to infinite

9.6 Constant – K Band pass filter:

- It is a filter which allow to pass of a limited band of frequencies (f_2-f_1) & attenuates (or) rejects all other frequencies below or above the frequency band..
- A band pass filter may be obtained by using a low pass filter followed by a high pass filter in which the cut- off frequency of the LP filter is above the cut- off frequency of the HP filter , the overlap thus allowing only a band of frequencies to pass .
- Consider the circuit in each arm has a resonant circuit with same resonant frequency, i.e. the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band pass characteristic.



f_1 = (K+$\sqrt{\boldsymbol{K^2} + \mathbf{L1}\ /\mathbf{C1}}$) /2π L_1

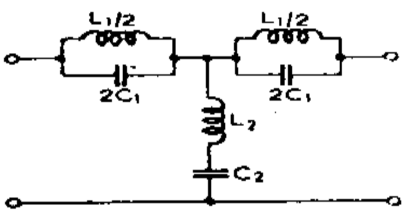
f_2 =- (K+$\sqrt{\boldsymbol{K^2} + \mathbf{L1}\ /\mathbf{C1}}$) /2π L_1

f_1 = lower cut-off frequency

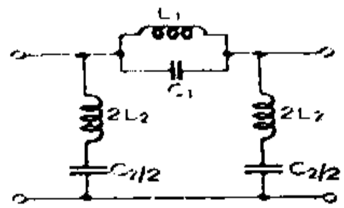
f_2 = higher cut-off frequency

9.7 Constant – K Band elimination filter:

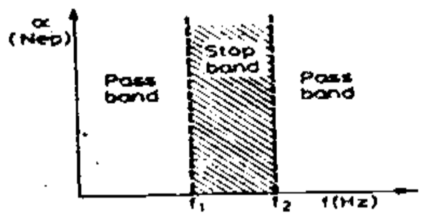
- This type of filter attenuates or rejects a limited band of frequencies but allow other frequencies
- A band elimination filter is one which passes without attenuation all frequencies less than the lower cut off frequency f1 , and greater than the upper cut off frequency f2 . Frequencies lying between f1 and f2 are attenuated. It is also known as band stop filter.



T-Configuration of BSF



π-Configuration of BSF



9.8 Solve Numerical problems:

Q-1 Design a high pass filter having a cut- off frequency of 1 kHz with a load resistance of 600 Ω.

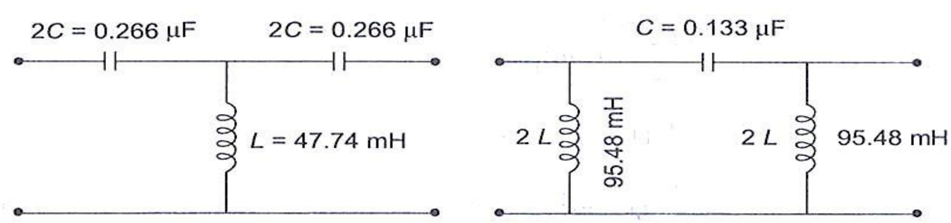
Solution.

It is given that $R_L = K = 600\ \Omega$ and $f_C = 1000$ Hz

$$L = \frac{K}{4\pi f_c} = \frac{600}{4\times\pi\times1000} = 47.74\,\text{mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{600}{4\times\pi\times600\times1000} = 0.133\,\mu\text{F}$$

The T and π– sections of the filter are shown in Fig.



(a) (b)

Problem: Design a LPF (both T and \pi networks) having a cut-off frequency of I kHz to operate with terminated load resistance of 200Ω Find the frequency at which this filter offers attenuation of 19. Idb.

Solution: Given

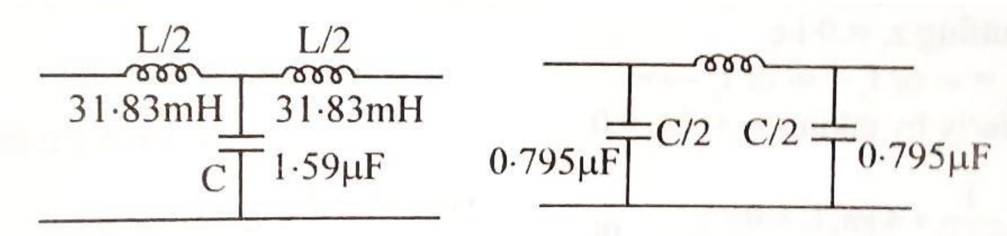
(i) $f_e = 1\text{kHz} = 1000$ Hz

$$R_0 = k = \sqrt{\frac{L}{C}} = 200\Omega$$

$\therefore L = \frac{R_0}{\pi f_c} = \frac{200}{\pi \times 1000} = 63.66mH$

$C = \frac{1}{\pi f_c k} = \frac{1}{\pi f_e R_0} = \frac{1}{\pi \times 1000 \times 200} = 1.59\mu F.$

The circuit diagram is given below :



(ii) Since the attenuation in db $= 8.666 \times$ Attenuation in nepers.

$\therefore$ Attenuation in neper $= \frac{19\cdot1}{8\cdot666} = 2 \cdot 2$Neper

For LPF : $\propto = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$

$\therefore 2.2 = 2\cosh^{-1}\left(\frac{f}{1}\right)$

$f = \cosh(1.1) = 1.67kHz.$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

Q-1 What is a band pass filter ? [W-14,17]

Ans: The filter which allows to pass of a limited band of frequency (f_1-f_2) and rejects all other frequencies below or above frequency.

Q-2 What is band stop filter ? [W-18]

Ans: The band stop filter blocks signals falling within a certain frequency band set up between two points while allowing both the lower and higher frequencies either side of this frequency band .

Q-3 What do you mean by filter ? Define pass band. [W-18,19]

Ans: A filter is an electrical network that can transmit signal within a specified frequency range. This frequency range is called as pass band.

Q-4 Classify filter ? [S-19, W-20]

Ans: Identifying their frequency characteristics, the filters are differentiated as,

1. Low pass filter (LPF)
2. High pass filter (HPF)
3. Band pass filter (BPF)
4. Band stop filter (BSF) or Band elimination filter (BEF)

POSSIBLE LONG TYPE QUESTIONS:

Q-1 Write short notes on constant K-low pass filter, constant K band pass filter [W-20]

Q-2 Explain constant K-high pass filter with diagram. [W-17]

Q-3 Design a m-derived T-section low pass filter having cut-off frequency of 7 K Hz, design impedance of 600 Ωs and frequency of infinite attenuation is 10.5 KHz. [S-18]

Q-4 Define active filter and explain different types of filters and draw their characteristics.[W-17]

Q-5 Classify filters and explain each filter with examples. [S-18,W-19]

Q-6 Design a band stop filter, constant K-filter with cut-off frequencies of 5Hz to 10KHz and nominal characteristic impedance of 300Ω.[W-14, W-18]